2025 Selection Test

for the International Physics Olympiad (IPhO), Asian Physics Olympiad (APhO) and International Nuclear Science Olympiad (INSO)

a. This is a **4 hour** test. Attempt all questions. The maximum total score is **80**; marks allocated for each question part are indicated in square brackets.

b. Check that there are a total of **13 printed pages** (*including* this cover page). The last page contains a table of physical constants that you may refer to and use.

c. Begin your answer for each question on a **fresh sheet of paper**, and present your working and answers clearly. Your answer sheets should be sorted according to the order of the questions.

d. Write your name on the **top right hand corner of every answer sheet** you submit.

e. You may use a standard (non-programmable) scientific **calculator** in accordance with the statutes of the International Physics Olympiad.

f. No external materials may be brought into the examination room. No discussion is allowed. Any intentional breach of integrity may lead to disqualification.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	6	5	5	10	10	14	10	12	8	80
Score:										

1. A wide river of uniform depth flows with a uniform constant speed u parallel to its banks. A boat is moving in the river with constant speed v, measured in the moving frame of the river.

   As the boat moves, a metal ball is dropped into the river with zero vertical velocity and the same horizontal velocity as the boat. The drag force law for the ball's motion in water is unknown. When the ball reaches the bottom of the river, the horizontal distance it has travelled since its point of release is measured in the stationary frame of the river bank.

   We will now consider three different cases, each with the boat moving in a different direction.

   (Case 1) When the boat is moving downstream (its net motion is parallel to the river velocity), the distance measured is a.

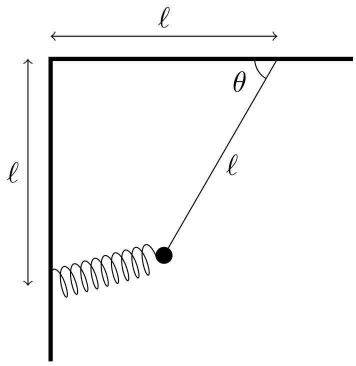
   (Case 2) When the boat is moving upstream (its net motion is antiparallel to the river velocity), the distance measured is b.

   (Case 3) When the boat is moving such that its net motion in the stationary frame is perpendicular to the velocity of the river, the distance measured is c.

   (a) Is it necessary to know the drag force law to determine the trajectory of the ball in the frame of the river? Explain your answer. *(Hint: Drawing a diagram may be helpful.)* [1]

   (b) With appropriate diagrams, find the ratio $\frac{v}{u}$. Leave your answer in terms of a, b and c. [5]

2. A pendulum with an inextensible string of length ℓ and mass m is attached to a spring of zero natural length and stiffness k. The string and spring are fixed to two perpendicular walls at distances l from the corner, as shown in the figure.

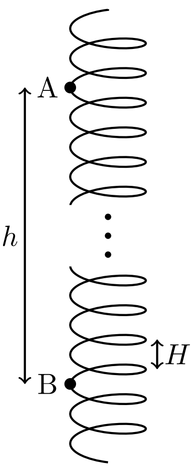


   (a) At equilibrium, $\theta = \theta_0$. Find θ_0. [2]

   (b) Find the angular frequency of small oscillations of the system about equilibrium. If required, leave your answer in terms of θ_0. [3]

3. A continuous rigid helix of uniform density has mass m and radius R. Its axis is oriented vertically, and the vertical distance between each helix turn is $H = \pi R$. The helix is able to rotate freely about its vertical axis but remains translationally fixed in place. It does not undergo compression nor extension.

   A small bead of identical mass m is threaded onto the frictionless helix. It is released from rest at point A and allowed to slide downwards along the helix.



   (a) Find the helix angle θ of the helix. The helix angle is the slope angle with respect to the horizontal, if the helix is unravelled. [1]

   (b) When the ball passes point B, located a vertical distance h directly below A, determine the angular velocity of the helix. If required, leave your answer in terms of θ. [4]

4. **Part A: Scaling Laws in a Column**

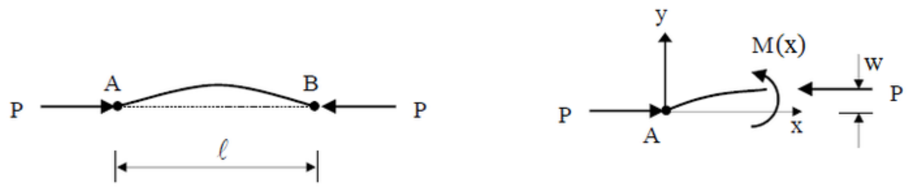
   Consider a solid cylindrical column of diameter d and height h supporting a sphere of diameter D on top. Assume that $D \gg d$, such that the contact area between the sphere and column is effectively the cross-sectional area of the column.

   (a) Suppose the diameter of the column is just sufficient to withstand the compressive load of the sphere. How should d scale with D, i.e. what should be the exponent α such that $d \propto D^{\alpha}$? *(Hint: The maximum stress that the solid column can withstand is a constant. Assume even stress across the contact area.)* [2]

Another possible mode of structural failure is buckling. According to the Euler-Bernoulli beam theory, the deflection w of a beam is related to its bending moment M by

$$M(x) = EI\frac{d^2w}{dx^2}$$

where E is the Young's modulus of the material (a constant), and $I = \int r^2 dA$ is the second moment of area about its central axis (analogous to the moment of inertia, but involving the cross-sectional area dA instead of the mass dm).

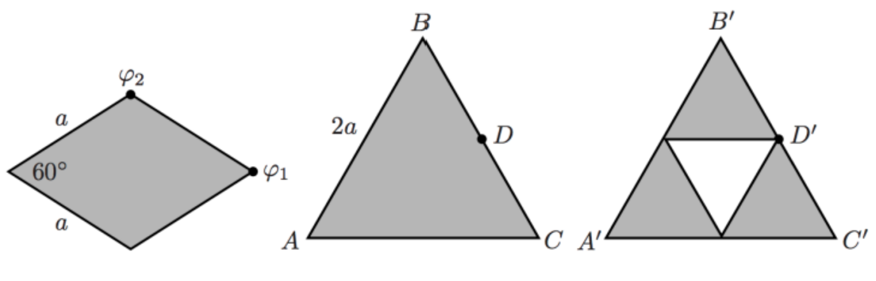


For example, in the figure above, a horizontal load P is applied inwards to both ends of a beam, causing deflections $w(x)$. The horizontal load causes a bending moment $M(x) = Pw(x)$. Above a critical load P_{crit}, the beam will undergo buckling.

(b) Consider the sphere-column system introduced in part (a). Given that $h \propto D$, how should d scale with D in order to prevent buckling? [3]

Part B: Scaling of Gravitational Potentials

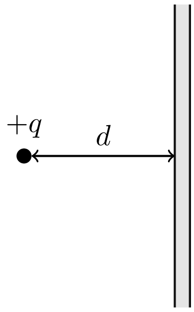
A massive thin rhombus plate, with side length a and acute apex angle 60°, has uniform surface mass density σ. The gravitational potential at the vertex of the acute angle of the rhombus is equal to φ_1, and the potential at the vertex of the obtuse angle is φ_2 (see first object in the figure).



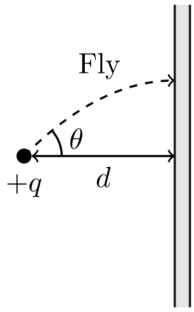
(c) An equilateral triangle of side length $2a$ (see second object in the figure) has the same uniform mass density σ. Find the gravitational potential at points C and D. Leave your answers in terms of φ_1 and φ_2. [3]

(d) An equilateral triangle of side a (see third object in the figure) is cut out from the center. Find the new potential at C' and D'. Leave your answer in terms of φ_1 and φ_2. [2]

5. Consider a point charge $+q$ placed at a fixed distance d from an infinitely large, thin, conducting plane.



A small fly, initially on the point charge, takes off with an initial angle θ from the horizontal. Flying at a constant speed v, it follows the path of an electric field line until it reaches the plane (this diagram is not drawn to scale).



(a) Define the coordinates (x, y) such that the plane is at $x = 0$, and the point charge is at $(-d, 0)$. Determine E_x and E_y, the x and y components of the electric field, for all points (x, y) in $x < 0$. [3]

(b) The electric field line illustrated terminates at $(0, y_0)$. Determine y_0, leaving your answer in terms of d and θ. (*Hint: The curved surface area of a sphere sector with half-angle θ is $\frac{1}{2}(1 - \cos\theta)$ of the total surface area of the sphere.*) [4]

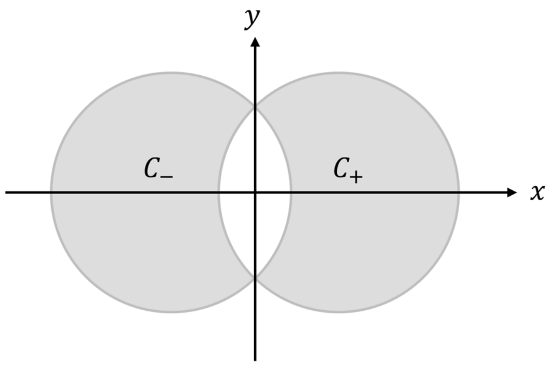
(c) Find the instantaneous acceleration of the fly a as it reaches the plane. Leave your answer in terms of v, d and θ. (*Hint: The radius of curvature R of a curve $y(x)$ is given by $R = \left|\frac{(1+y'^2)^{\frac{3}{2}}}{y''}\right|$, where primes denote a derivative with respect to x.*) [3]

6. Superconductors exhibit the Meissner effect where below a critical temperature, the superconducting material expels all magnetic fields from its interior. This effect can be visualised by magnetic field lines, which are unable to penetrate the superconducting surface, instead curving around the superconductor.

   This behaviour is similar to fluid flow around a solid object, and we may draw an analogy between electromagetism and fluid dynamics. In this problem, we will exploit this analogy to discuss the Magnus effect; a phenomenon that occurs when a rotating object moves through a fluid.

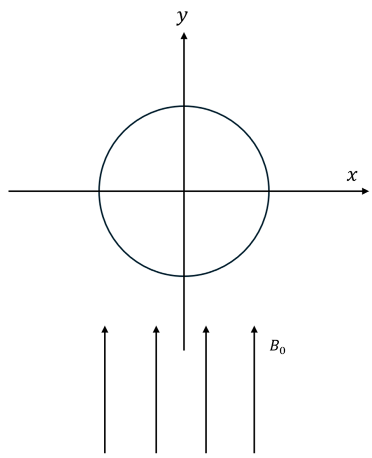
   (a) Consider an infinitely long conducting cylinder of radius R, carrying current I along its axis of symmetry distributed uniformly across its cross section. Find $B(r)$ for $r < R$. [2]

   Two straight, infinitely long cylindrical nonmagnetic conductors C_+ and C_-, insulated from each other, overlap. They carry uniformly distributed current I in and out of the paper respectively (i.e. C_+ carries a current of I out of the page and C_- carries a current of I into the page. The overlapping region has zero net current). The cross sections of the conductors (shaded in the figure) are limited by circles of radius R in the x-y plane, with distance d between their centres.



   (b) Determine the magnetic field $\vec{B}(x, y)$ in the space between the conductors. The origin is placed in the middle of the two centres. [2]

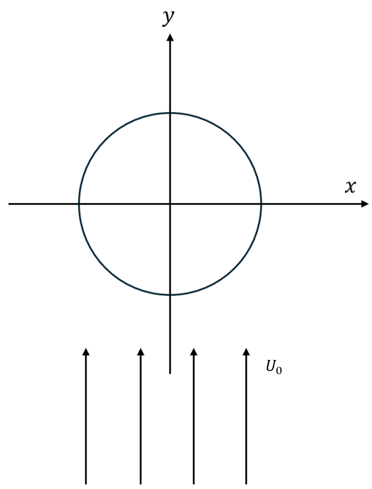
   An infinite solid superconducting cylinder of radius R with symmetry axis parallel to the z-axis lies in a uniform external magnetic field of magnitude B_0 parallel to the y-axis.



   (c) Knowing that superconductors repel magnetic fields, and using the result in part (b), show that the net magnetic field $\vec{B_1}$ in the region $r > R$ is given by: [3]

$$\vec{B_1}(r, \theta) = B_0 \sin\theta \left(1 - \frac{R^2}{r^2}\right) \hat{r} + B_0 \cos\theta \left(1 + \frac{R^2}{r^2}\right) \hat{\theta}$$

An infinitely long solid cylinder of radius R is placed in a region of incompressible, non-viscous fluid that flows from $y = -\infty$ with a uniform velocity of $U_0\,\hat{y}$, past the cylinder and away towards $y = +\infty$. Consider the fluid motion in one cross-sectional plane of the cylinder.



(d) Draw a diagram to represent the fluid flow around the cylinder and write down an expression for $\vec{U}(r,\theta)$, the velocity of the fluid at any point in space outside the cylinder in polar coordinates, justifying your answer. [2]

The cylinder is now given an angular velocity that points in the positive z-direction (out of the paper), inducing circular currents in the fluid around it. To model the effect of the rotation of the cylinder on the surrounding fluid, we will use the concept of circulation.

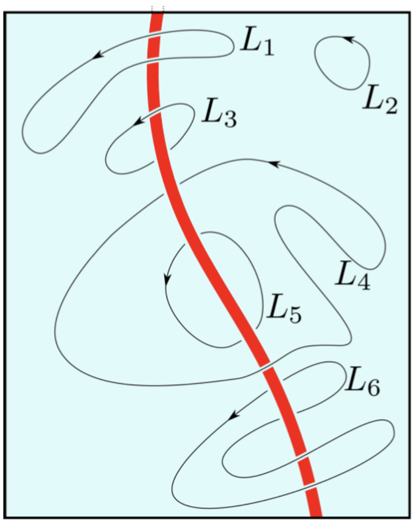
Usually, non-viscous flow has an important property of being irrotational: the circulation of velocity along any closed path within the fluid is zero.

$$\oint \vec{v} \cdot d\vec{l} = 0$$

However, this changes if we introduce a *vortex filament*; which induces long range circulatory flows in the fluid. For any closed loop that wraps around these filaments,

$$|\oint \vec{v} \cdot d\vec{l}| = 2\pi\Gamma$$

where Γ is called the circulation quantum. To illustrate this, a vortex filament (thick line) is drawn in fluid. The velocity circulation along paths L_1, L_2, L_5 and L_6 (thin lines) are all zero, whereas those for L_3 and L_4 are equal to $\pm 2\pi\Gamma$. Note that circulations along L_3 and L_4 have opposite signs.

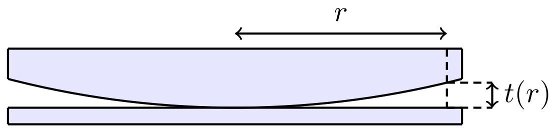


(e) Model the rotation of the cylinder with a long infinite vortex filament with circulation quantum Γ placed along the central axis of the cylinder. Find the new velocity field $\vec{U}_1(r, \theta)$. [2]

(f) Given density of fluid ρ, find the force per unit length $\vec{F}$ acting upon the cylinder due to the fluid flow. *(Hint: One possible solution is to apply Bernoulli's equation.)* [3]

7. **Part A: Thin Lens Interference**

A plano-convex lens with radius of curvature R and refractive index n is placed on top of a flat glass plate such that there is a small air gap between the curved surface and the plate.

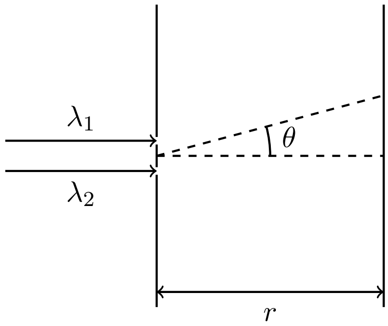


(a) Consider the thickness $t(r)$ of the air gap as a function of radial distance r from the centre of the lens. Determine $t(r)$ to leading order in r. [1½]

(b) Collimated light of wavelength λ is incident on the lens. Determine the the radial position r_m of the m^{th} bright fringe. Assume that the incident light and reflected light is always approximately normal to both the lens and plate. [2½]

Part B: Double Slit Diffraction

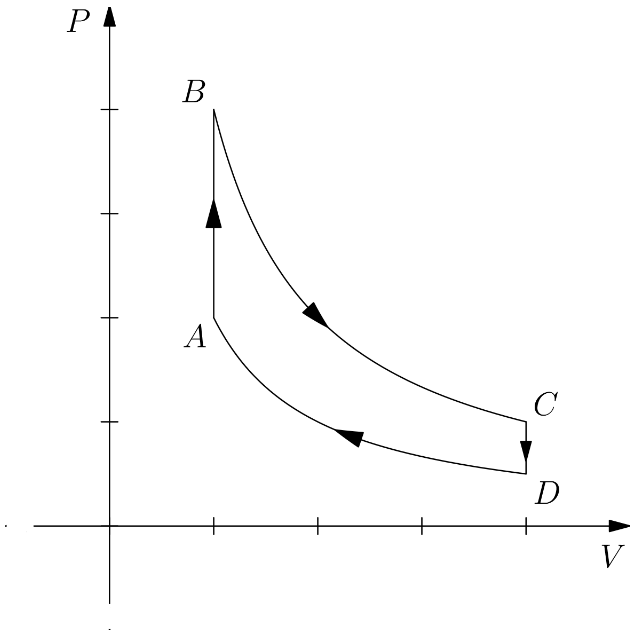
We consider two-slit diffraction, but with collimated light of speed c and two different wavelengths λ_1, λ_2 entering the two slits respectively. The two slits are separated by distance d. For simplicity, we project onto a screen placed a distance r away, and define the angular position θ at the screen as shown in the figure. Assume $r \gg d \gg \lambda_1, \lambda_2$. Set the phases of the two waves at the slit to be $\phi = 0$.



(c) Let the electric field through each slit have amplitude E_0. Find the amplitude of the total electric field at an angle θ on the screen, at an arbitrary time t. Express your answer as a product of cosines or sines. *(Hint: You may use the identity* $\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$*. You may choose to define your own variables to simplify your final answer.)* [3]

(d) Now suppose $\lambda_1 = \lambda - \delta\lambda, \lambda_2 = \lambda + \delta\lambda$, where $\delta\lambda \ll \lambda$. Consider the time average to be taken over the period corresponding to λ. Find the intensity pattern $I(\theta, t)$. State the angles which correspond to maxima and minima, and comment on how the positions of these maxima and minima near $\theta = 0$ change with time. You may use appropriate approximations. [3]

8. This is the PV-diagram of the Otto cycle: $A \to B$ and $C \to D$ are isochoric (or isovolumetric) processes, and $B \to C$ and $D \to A$ are adiabatic processes.



(a) Consider a classical Otto engine, containing a gas with adiabatic index γ with volumes V_1 and V_2 respectively at the two isochoric processes ($V_1 < V_2$). Determine the efficiency η_c of such an engine. [3]

Now, we will discuss the quantum Otto engine. For simplicity, consider a two-level atomic system with ground state and excited state energies $E_0 = 0$ and E_1, and suppose the probabilities of existing in these states are p_0 and p_1 respectively. The average energy of the two-level atom is thus

$$\langle E \rangle = p_0 E_0 + p_1 E_1 = p_1 E_1$$

Suppose the energy difference E_1 between the two states can be adjusted throughout the cycle. The probability p that the atom is in the energy state E satisfies the Boltzmann distribution

$$p \propto e^{-E/(k_B T)}$$

where T is the temperature and k_B is the Boltzmann constant.

(b) When the quantum matter is in equilibrium with a heat reservoir of temperature T, write down expressions for the probabilities p_0 and p_1. Leave your answers in terms of E_1, k_B and T. [1]

In a thermodynamical process, the energy change dE can be written in terms of the change in work and heat using the First Law of Thermodynamics.

$$dE = dW + dQ$$

In a quasi-static quantum process, the change in average energy is given by differentiating the equation for average energy.

$$d\langle E \rangle = p_1 \, dE_1 + E_1 \, dp_1$$

The quantum adiabatic theorem states that the probabilities of each quantum state remain effectively constant during an adiabatic process.

(c) Write an equation for $d\langle E\rangle$ in an adiabatic process. Leave your answer in terms of p_1, E_1 and their differentials. [1]

The von Neumann entropy S is given by

$$S = -k_B \sum_i p_i \ln p_i$$

where k_B is the Boltzmann constant and p_i is the probability of the i-th state in the quantum system.

(d) Using the equation for von Neumann entropy, write an equation for $d\langle E\rangle$ in an isochoric process. Leave your answer in terms of p_1, E_1 and their differentials. [2½]

(e) Sketch the quantum Otto cycle on the axes E_1 against p_1, with arrows and labels for A, B, C, D. You may use the first law of thermodynamics, or your results from parts (c) and (d). [2½]

(f) Compute the efficiency η_q of the quantum Otto cycle, in terms of the temperatures T_B and T_C at the states B and C respectively. [2]

9. When stars collapse, over 97% of them become white dwarves. These are extremely dense bodies consisting largely of degenerate electron matter and some ions. Unlike stars, the white dwarves can no longer support itself against gravitational collapse by its gas pressure, and instead the electron degeneracy pressure dominates.

For the free electrons in white dwarves, we must use a quantum mechanical description. We can define their number density of state $g(p)$ as:

$$g(p)\,dp = \frac{8\pi}{h^3}p^2\,dp$$

where p is the momentum of the state and h is Planck's constant. In other words, in a volume dV, there are $g(p)\,dp\,dV$ states of momentum p that may be occupied by an electron.

To describe the probability in which these states are occupied, we can apply Fermi-Dirac statistics, which tells us that a state of energy ε has an average occupation probability $f(\varepsilon)$:

$$f(\varepsilon) = \frac{1}{\exp\left(\frac{\varepsilon-\mu}{kT}\right)+1}$$

where k is the Boltzmann constant, T is the temperature and μ is an energy term known as the chemical potential.

(a) Show that the pressure P due to the free electrons is given by: [2]

$$P = \frac{8\pi}{3h^3}\int_0^\infty \frac{p^3}{\exp\left(\frac{\varepsilon-\mu}{kT}\right)+1}v(p)\,dp$$

where $v(p)$ is the magnitude of the velocity of an electron as a function of its momentum p.

We now make the assumption that the electron gas is fully degenerate. This means that all the electron states are occupied up till the state with Fermi momentum p_f, and no electron states above that are occupied. This is equivalent to assuming zero temperature for the gas, and the energy of the electron is equal to μ when it occupies the state with momentum p_f.

(b) Under this assumption, show that the pressure P can be simplified to: [1½]

$$P = \frac{8\pi}{3h^3}\int_0^{p_f} p^3 v(p)\,dp$$

(c) Determine an expression for n_e, the number density of electrons. Assume that the electron gas is fully degenerate. Leave your answer in terms of h and p_f. [1½]

(d) Assume that the electrons only move non-relativistically. Determine the pressure P for the electron cloud. Leave your answer in terms of h, m_e and n_e. [1½]

Unlike the electrons in the white dwarf, the ions can be described using classical ideal gas equations. Consider the white dwarf Sirius B, which we assume to be purely carbon such that the number of ions n_i is given by $6n_i = n_e$. The electrons here move non-relativistically.

(e) Estimate the numerical ratio of pressures exerted by the electrons to the ions $\frac{P_e}{P_i}$. The mean density of Sirius B is 2.38×10^9 kg m^{-3} and its temperature can be estimated to be 25000 K. Despite the non-zero temperature, assume that your result in part (d) remains valid. [1½]

Fundamental Physical Constants — Frequently used constants

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c	299 792 458	m s^{-1}	exact
Newtonian constant of gravitation	G	$6.674\,30(15) \times 10^{-11}$	m^3 kg^{-1} s^{-2}	2.2×10^{-5}
Planck constant*	h	$6.626\,070\,15 \times 10^{-34}$	J Hz^{-1}	exact
	$\hbar$	$1.054\,571\,817\ldots \times 10^{-34}$	J s	exact
elementary charge	e	$1.602\,176\,634 \times 10^{-19}$	C	exact
vacuum magnetic permeability $4\pi\alpha\hbar/e^2c$	μ_0	$1.256\,637\,061\,27(20) \times 10^{-6}$	N A^{-2}	1.6×10^{-10}
vacuum electric permittivity $1/\mu_0c^2$	ϵ_0	$8.854\,187\,8188(14) \times 10^{-12}$	F m^{-1}	1.6×10^{-10}
Josephson constant $2e/h$	K_J	$483\,597.848\,4\ldots \times 10^9$	Hz V^{-1}	exact
von Klitzing constant $\mu_0c/2\alpha = 2\pi\hbar/e^2$	R_K	$25\,812.807\,45\ldots$	Ω	exact
magnetic flux quantum $2\pi\hbar/(2e)$	Φ_0	$2.067\,833\,848\ldots \times 10^{-15}$	Wb	exact
conductance quantum $2e^2/2\pi\hbar$	G_0	$7.748\,091\,729\ldots \times 10^{-5}$	S	exact
electron mass	m_e	$9.109\,383\,7139(28) \times 10^{-31}$	kg	3.1×10^{-10}
proton mass	m_p	$1.672\,621\,925\,95(52) \times 10^{-27}$	kg	3.1×10^{-10}
proton-electron mass ratio	m_p/m_e	$1836.152\,673\,426(32)$		1.7×10^{-11}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5643(11) \times 10^{-3}$		1.6×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,177(21)$		1.6×10^{-10}
Rydberg frequency $\alpha^2m_ec^2/2h$	cR_∞	$3.289\,841\,960\,2500(36) \times 10^{15}$	Hz	1.1×10^{-12}
Boltzmann constant	k	$1.380\,649 \times 10^{-23}$	J K^{-1}	exact
Avogadro constant	N_A	$6.022\,140\,76 \times 10^{23}$	mol^{-1}	exact
molar gas constant N_Ak	R	$8.314\,462\,618\ldots$	J mol^{-1} K^{-1}	exact
Faraday constant N_Ae	F	$96\,485.332\,12\ldots$	C mol^{-1}	exact
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,374\,419\ldots \times 10^{-8}$	W m^{-2} K^{-4}	exact
Non-SI units accepted for use with the SI				
electron volt (e/C) J	eV	$1.602\,176\,634 \times 10^{-19}$	J	exact
(unified) atomic mass unit $\frac{1}{12}m(^{12}\text{C})$	u	$1.660\,539\,068\,92(52) \times 10^{-27}$	kg	3.1×10^{-10}

* The energy of a photon with frequency ν expressed in unit Hz is $E = h\nu$ in J. Unitary time evolution of the state of this photon is given by $\exp(-iEt/\hbar)|\varphi\rangle$, where $|\varphi\rangle$ is the photon state at time $t = 0$ and time is expressed in unit s. The ratio $Et/\hbar$ is a phase.