Singapore Physics Olympiad Training

2023 Selection Test
for the Asian and International Physics Olympiads

Marker's Report

1 Question 1

The essence of this question was to analyse the motion in the CM frame. However, there was some confusion among students regarding the notation used.

(a) Many students did not read that v_f and v_i were referring to the centre of mass speeds. Several students misunderstood that both masses are reflected elastically by the wall (which would not produce any oscillations!). Others simply stated an elastic collision implied $e = 1$.

Less than a third of the students got this part correct.

(b) The most common error was taking the spring constant to be $2k$ although the question specified the effective spring constant of the rod was k. Many students found the correct idea of going into the CM frame. Stating that a spring force being negatively proportional to displacement is not enough, since it does not demonstrate oscillations about the CM.

(c) Students who got to this point generally answered well. The most common mistake was made during the computation of the amplitude of oscillation, where students took $x = \frac{v_i}{\omega}$ instead of $\frac{2v_i}{\omega}$.

The intention of this question was to demonstrate that the coefficient of restitution need not be 1 in an elastic collision. However, there is a small technical error in the question, which is that if the two masses are close enough, the oscillation of m_1 may cause it to "phase" into the wall. Namely, since the CM speed is $v_{cm} = \frac{m_1 - m_2}{m_1 + m_2} v_i$ to the left, the position of the CM is

$$x_{cm} = -\frac{m_1 - m_2}{m_1 + m_2} v_i t.$$

In the CM frame, the motion of m_1 satisfies

$$x_1 = -\frac{2m_2 v_i}{m_1 + m_2} \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} \sin\left(\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} t\right).$$

Therefore, it might be the case where $x_1 + x_{CM} > 0$:

$$x_1 + x_{CM} = -\frac{m_1 - m_2}{m_1 + m_2} v_i t - \frac{2m_2 v_i}{m_1 + m_2} \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} \sin\left(\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} t\right) > 0$$

$$-2m_2 \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} \sin\left(\sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} t\right) > (m_1 - m_2) t$$

This is a transcendental equation, however we can still obtain a condition on m_1 and m_2 where it hits the wall somewhere in the motion. Suppose at the amplitude of the oscillation, $x_1 + x_{CM} > 0$. Then

$$t = \frac{3\pi}{2} \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$

$$2m_2 \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}} > (m_1 - m_2) \cdot \frac{3\pi}{2} \sqrt{\frac{m_1 m_2}{k(m_1 + m_2)}}$$

$$\frac{4}{3\pi} > \frac{m_1 - m_2}{m_2}$$

$$\frac{m_1}{m_2} < \frac{4}{3\pi} + 1$$

You could try to obtain a more exact bound by plotting a graph of $y = -2\sin x$ against $y = \alpha x$ and see where the curves are tangent, from there the value of α will tell give you the maximum ratio of $\frac{m_1}{m_2}$ such that a collision will occur (in fact, $\alpha \approx 0.44$ so the maximum ratio is around 1.44.)

2 Question 2

This is known as the Child-Langmuir Law and is adapted from a problem in [1].

(a) Almost all students managed to use COE correctly. Students who didn't get this part used V as a linear function of x (which isn't true as we will find out later in the question).

(b) Most students were able to use Gauss' Law in conjunction with $I = A\rho v$.

(c) Few students were able to solve the ODE in the previous part. Those who did so either used an ansatz $V = \alpha x^n$, or made use of $V'' = \frac{dV'}{dx} = \frac{dV'}{dV}\frac{dV}{dx} = V'\frac{dV'}{dV}$.

Some students made use of $\int V^{1/2} d^2V = \int \frac{2}{3} V^{3/2} dV = \frac{4}{15} V^{5/2}$. This is wrong because the first integral is actually $\int V^{1/2} dV'$, which is different from $\int\int V^{1/2} dV\, dV$ (i.e. d^2V is not the same as dV^2).

(d) Those who managed to solve (or get an expression) for the previous part mostly managed to use $V = V_0$ at $x = d$.

3 Question 3

This question was pitched at a level that only involves elementary theory from electromagnetism. However, many students were uncertain about the transformation of electromagnetic fields between frames. A couple of students also only stated the magnitude of the electromagnetic field without stating the equally important direction, resulting in an unnecessary loss of marks.

(a) Almost all students got this correct.

(b) Everyone managed to state the correct formula, but a handful took the wrong velocity. Some also only gave R_1 and not T even though the question asked for both.

(c) Almost all students understood how Bob was moving and got this part.

(d) Those who got the previous part got this as well.

(e) A common misconception is that the electric field acts in the drift direction. This isn't actually the case; the correct method is to balance the Lorentz forces.

(f) Regrettably, only half the scripts managed to deduce that Carol was stationed at the centre of the circle and rotating at the same angular velocity as P_1.

(g) Those who got up to this point generally managed to get the correct magnetic field.

4 Question 4

This question was adapted from Romanian Masters of Physics 2017, and demonstrates a "paradox" in which angular momentum is seemingly not conserved. However, it is in fact stored within the electromagnetic field that is radiating away.

(a) Everyone managed to solve this part. Well done!

(b) Those who understood the mechanism behind how the cylinders rotated progressed far in this problem. Basically, it is due to the induced emf from the rotating solenoid that creates an electric field, which then generates a torque on each of the cylinders. Some students went down a rabbit-hole of considering the rotating cylinders having surface current due to them being charged and generating their own B-field.

(c) The resolution to this paradox is that the electromagnetic fields can carry angular momentum. The handful of students who saw this and calculated the Poynting vector (and hence the linear momentum density) managed to obtain the quantities of the mechanical angular momentum and the EM field angular momentum and show that they are equal.

5 Question 5

This question tested students' understanding of impedance as complex numbers. Those who were familiar with the concept managed to work through most of the question, while those who did not realise the properties of the numbers in question struggled with making headway.

(a) Those who knew the property of complex impedances of the three elements managed to find the source voltage easily.

(b) Regrettably, although many students found that Z_1 was a resistor, they did not realise that Z_2 and Z_3 could be interchanged, and just wrote down one combination for them (instead of writing that we cannot tell which is which – the question is worded "determine the possible identities" so the presence of more than one case is acceptable)

(c) Most students who got the first part managed to do this. There were, however, several algebra mistakes which resulted in a ratio that was wrong.

6 Question 6

This question on thermodynamic identities was generally well done.

(a) Most students were able to use the first law of thermodynamics in conjunction with the ideal gas law (in differential form).

(b) The intended method was to write the 1st law of thermodynamics in terms of infinitesimal entropy change, equate to zero, and compare this with the adiabatic expression. Several students found other (quicker) methods, such as finding c_p and c_v, or simply noting that

$$\frac{dU}{dT} = (n_1 + n_2)c_v = n_1 c_{v1} + n_2 c_{v2}$$

and using $\gamma = \frac{c_p}{c_v}$.

7 Question 7

This question was adapted from IPhO 2004 T2.

(a) A number of students made errors in their units, for example, taking $P = F_T/r$. This caused outright wrong expressions (and led to no maximum value in the next part). However, some students made some subtle errors like dividing the force by πr^2 which would yield an answer which is off by a factor of 4. The intended approach was to find the work done (and hence change in internal energy) to inflate the balloon by a radius dr, from which one can find the pressure.

(b) This part was generally well done by those who got somewhere in the previous part (since the numerical prefactor doesn't matter in this case). Students either differentiated with respect to r or λ, which yielded equivalent results.

(c) Majority of students were able to find the radius of the unstretched balloon using the ideal gas law and the ratio of initial to final pressure. Almost all who did went on to solve this part fully by using the ideal gas law with the other results from the previous parts.

8 Question 8

This question is on pulse dispersion in optical fibres, specifically modal dispersion. For a more in-depth read, check out [2]. The mathematics in the question was the tricky part, especially the integral in (d)(i) [in fact, this question is the only one in the paper without a perfect script!].

(a) Almost all students were able to complete this part, either by differentiating $(dr/dz)^2$ with respect to z, or just expressing dr/dz in terms of n and β and differentiating from there.

(b) Most students were able to draw the analogy between this and simple harmonic motion, resulting in a sinusoidal equation. However, some students stumbled on getting the initial conditions right, or finding the value of z_1 (which occurs when the argument of the sine function is π).

The students who completed (b)(i) were able to make the approximation $n_1 \approx \beta$.

On the other hand, (b)(ii) could be tackled by using β directly (i.e. students do not need to utilise the previous part).

(c) This part was well done. Students were able to express the arc length in terms of n and β and form the required integral.

(d) This part tested students' mathematical capability in evaluating integrals. The integral in question, however, proved to be rather challenging – a couple of students put up a valiant effort, but no one was able to complete the part fully.

(e) This part uses the result from the previous part. Since $n_2 < \beta < n_1$, we may use these to be the maximum and minimum times.

(f) This question is unrelated to the previous four parts, and can be solved by just using preoperties of total internal reflection. There were only a handful of correct solutions, though – students were probably scared off by the earlier parts or ran out of time.

References

[1] David J. Griffiths. *Introduction to Electrodynamics*. Cambridge University Press, 2017.

[2] Ajoy Ghatak; K. Thyagarajan. *An Introduction to Fibre Optics*. Cambridge University Press, 1998.