



Singapore Physics Olympiad Training

2022 Selection Test

for the Asian and International Physics Olympiads

1. This is a **four-hour** test. Attempt all questions. The maximum total score is **90**; marks allocated for each question part are indicated in square brackets.
2. Check that there are a total of **11 printed pages** (*including* this cover page). The last page contains a table of physical constants that you may refer to and use.
3. Begin your answer for each question on a **fresh sheet of paper**, and present your working and answers clearly. Your answer sheets should be sorted according to the order of the questions.
4. Write your name on the **top right hand corner of every answer sheet** you submit.
5. Please **complete and sign the declaration on page 2**, which should be **stapled together and submitted** with your answer sheets.
6. You may use a standard (non-programmable) scientific **calculator** in accordance with the statutes of the International Physics Olympiad.
7. No books or documents relevant to the test may be brought into the examination room.

Declaration

I declare that I will be fully committed to the training for and participation in the Asian Physics Olympiad and/or the International Physics Olympiad if selected. I will check first with the MOE coordinator before taking on **additional commitments not listed below**.

Potential limitations to my commitment in the period from now to end-July 2022 are **described exhaustively** in the box below, such as other academic competitions, CCA commitments (school-related or otherwise), travel plans, etc.

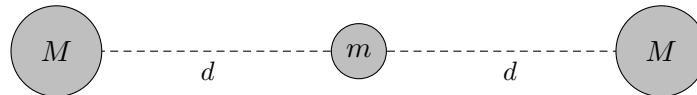
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Question:	1	2	3	4	5	6	7	8	9	Total
Points:	5	7	12	13	13	8	11	9	12	90
Score:										

1. Suppose that masses m_1 and m_2 separated by a distance r have an interaction potential energy given by

$$U = \frac{\kappa m_1 m_2}{r^n}, \text{ where } \kappa > 0 \text{ and } n \text{ is a positive integer.}$$

In this problem, consider a mass m confined to move along the line segment of length $2d$ between two identical particles of mass M . Assume that the two masses M are fixed in position.



- (a) State, with brief reasons, whether the interaction is an attractive or a repulsive force. [1]
 (b) Determine, in terms of the symbols introduced, the angular frequency ω of small oscillations of m around its equilibrium position (i.e. for displacement $x \ll d$). [4]

Q1 total: 5

2. Consider a yo-yo made up of two uniform solid disks of radius R and each with mass M , connected rigidly by a light cylindrical axle of radius $r < R$, such that the disks and axle all share a common axis. A thin light string is wound tightly around the axle.

The free end of the string is held fixed and the yo-yo is released from rest. Assume that the string stays vertical as it is unwound from the axle. Let g be the gravitational acceleration.

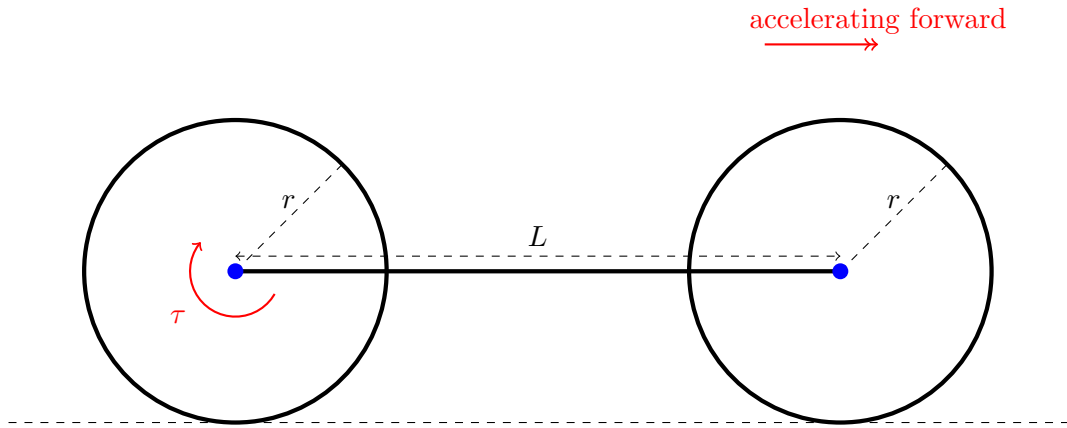
- (a) By integrating over thin circular rings, show that the moment of inertia I of the yo-yo about its central axis is given by $I = MR^2$. [2]
 (b) Determine, in terms of the symbols introduced, the downward acceleration a of the yo-yo when it is released from rest. [4]
 (c) Determine, in terms of the symbols introduced, the tension T in the string when the yo-yo is released from rest. [1]

Q2 total: 7

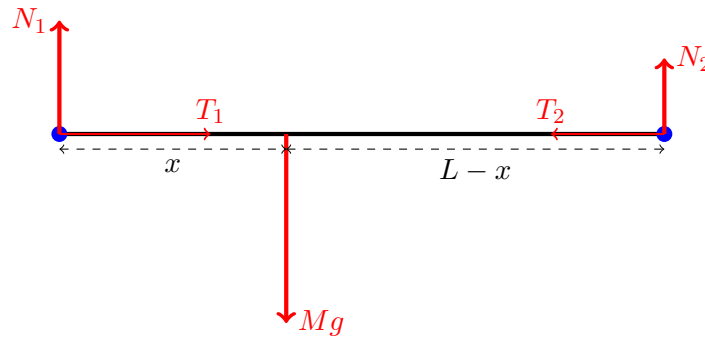
3. In this problem, we consider a minimal mechanical model of a bicycle moving along a horizontal surface.

Model the wheels as two uniform solid disks of radius r , each of mass m and moment of inertia I about their centres. The wheels can rotate around their central axes and these axes are connected by a rigid bar of length L and mass M . This bar represents the bicycle frame and rider, and we assume the bar is always horizontal with the centre of mass of this bar located a distance x from the rear wheel (and thus a distance $L - x$ from the front wheel).

The rider exerts a pure torque τ on the rear wheel. The entire bicycle has linear acceleration a in the forward direction. Assume that the wheels do not slip.



The free-body diagram of the bar (representing the frame and rider) is shown below. N_1 and T_1 are the vertical and horizontal forces respectively exerted by the rear wheel on the bar, while N_2 and T_2 are exerted by the front wheel on the bar.



- (a) Write down an expression for acceleration a in terms of T_1, T_2 and M . [1]
- (b) Write down an expression for N_2 in terms of N_1 and Mg . [1]
- (c) Draw separate free-body diagrams for each of the rear and front wheels. Label the normal contact and frictional contact forces with the ground as $R_1 = N_1 + mg$ and f_1 respectively for the rear wheel, and $R_2 = N_2 + mg$ and f_2 for the front wheel. [3]
- (d) Determine an expression for a in terms of τ, r, m, M and I . [4]
- (e) Suppose the frictional forces are maximal, i.e. $f_{1,2} = \mu R_{1,2}$ where μ is the coefficient of static friction (assumed to be the same for both wheels). Determine an expression for the ratio R_1/R_2 in terms of r, m, M and I and show that $R_1/R_2 > 1$. [3]

Q3 total: 12

4. Starting from Maxwell's equations for electromagnetism, we can identify the energy density (i.e. energy per unit volume of space) in an electromagnetic field as

$$u = \frac{\epsilon_0}{2} \vec{E} \cdot \vec{E} + \frac{1}{2\mu_0} \vec{B} \cdot \vec{B},$$

where ϵ_0 is the vacuum electric permittivity, μ_0 is the vacuum magnetic permeability, and \vec{E}, \vec{B} are the electric and magnetic field vectors respectively.

Poynting showed that the energy flow \vec{S} corresponding to changes in this energy density u is given by a vector cross product,

$$\vec{S} = \epsilon_0 c^2 \vec{E} \times \vec{B},$$

which we now term the Poynting vector. In this expression, c is the speed of light in vacuum. The integral of \vec{S} over a closed surface gives the total energy flow in or out of the enclosed volume.

- (a) Consider a propagating light wave with wavelength λ , given by

$$\begin{cases} \vec{E}(x, t) = E_0 \cos \left[\frac{2\pi}{\lambda} (x - ct) \right] \hat{y} \\ \vec{B}(x, t) = \frac{E_0}{c} \cos \left[\frac{2\pi}{\lambda} (x - ct) \right] \hat{z} \end{cases}$$

where E_0 is the amplitude of the electric field and \hat{y}, \hat{z} are the unit vectors in the y -direction and z -direction respectively.

- i. Determine an expression for $\langle E^2 \rangle$, the time-averaged value of $\vec{E} \cdot \vec{E}$. [1]

- ii. Using the relation $c = 1/\sqrt{\epsilon_0 \mu_0}$, show that the time-averaged value of the energy density u is given by [2]

$$\langle u \rangle = \epsilon_0 \langle E^2 \rangle .$$

- iii. State the direction of \vec{S} and show that the time-averaged magnitude $S = |\vec{S}|$ is given by [2]

$$\langle S \rangle = c \langle u \rangle .$$

Note that this is consistent with the interpretation of \vec{S} as the energy flow due to the propagation of light.

- (b) Now consider a cylindrical section of conducting wire, with resistivity ρ , length L and radius r . Let the current through the wire be I .

- i. The magnetic field pattern is concentric around the wire. Write down, in terms of the symbols provided, the expression for $B = |\vec{B}|$ at the curved surface of the wire. [1]

- ii. Determine an expression for the potential difference V between the two ends of the wire in terms of the symbols provided. [2]

- iii. Assume that the electric field in the wire is uniform. State the direction of \vec{S} and determine an expression for $S = |\vec{S}|$ at the curved surface of the wire. [3]

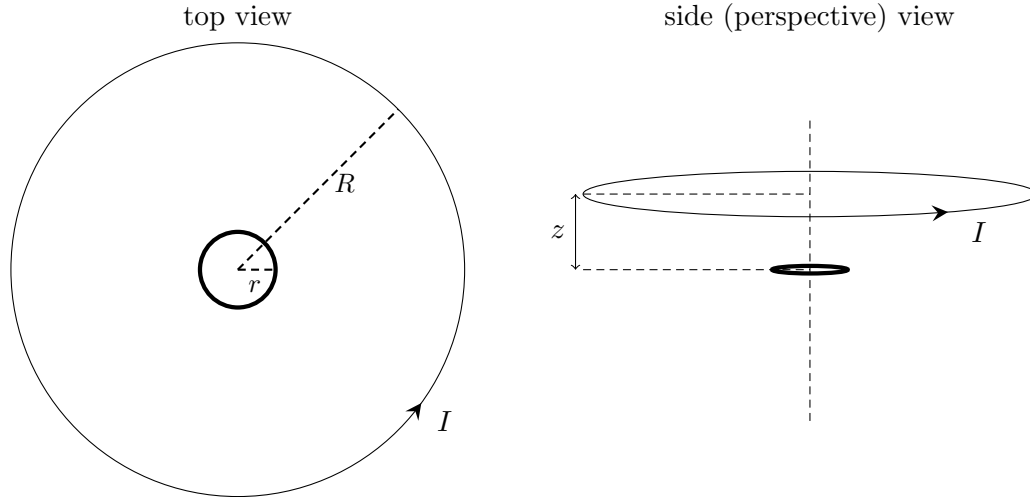
- iv. Hence determine an expression for the power P to the wire. [2]

Q4 total: 13

5. Two thin rigid circular loops share a common axis as shown. We can ignore the effects of gravity in this question.

The larger loop is a conducting wire of radius R . This wire carries a constant current I , and slides up the axis at constant speed $v \ll c$ so that the distance z between the two loops increases linearly with time t .

The other loop is a smaller insulating ring of radius $r \ll R$, with a uniform positive linear charge density of $+\lambda$ and a uniform linear mass density of ρ . The smaller loop can rotate freely around the central axis but cannot move up and down the axis.



At time $t = 0$, the loops are co-planar (i.e. $z = 0$) and the smaller loop is not rotating (i.e. angular speed $\omega = 0$).

- (a) Briefly explain why the smaller loop starts spinning and whether the orientation of this rotation is in the same or opposite sense as the current I in the larger loop. [2]
- (b) The current I in the larger loop produces a magnetic field. Using the Biot-Savart law, show that at the centre of the smaller loop, the magnetic flux density due to this current is directed upwards along the axis and has a z -dependence given by [3]

$$B_I(z) = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}, \quad (1)$$

where μ_0 is the permeability of free space.

- (c) The rotation of the smaller loop also contributes to the magnetic flux density at the centre of the smaller loop. Determine an expression, in terms of μ_0 , λ , r and the angular speed ω , for this rotational contribution $B_\lambda(\omega)$. [3]
- (d) Assume that the magnetic field within the entire area of the smaller loop is approximately constant and is the same as the field at the centre of the loop, such that the resultant magnetic flux density is $B(z, \omega) = B_I(z) + B_\lambda(\omega)$. [5]

Determine the angular speed ω_∞ at long times $t \rightarrow \infty$, in terms of μ_0 , I , λ , ρ , r and R .

Q5 total: 13

6. For a certain circuit component shown below,



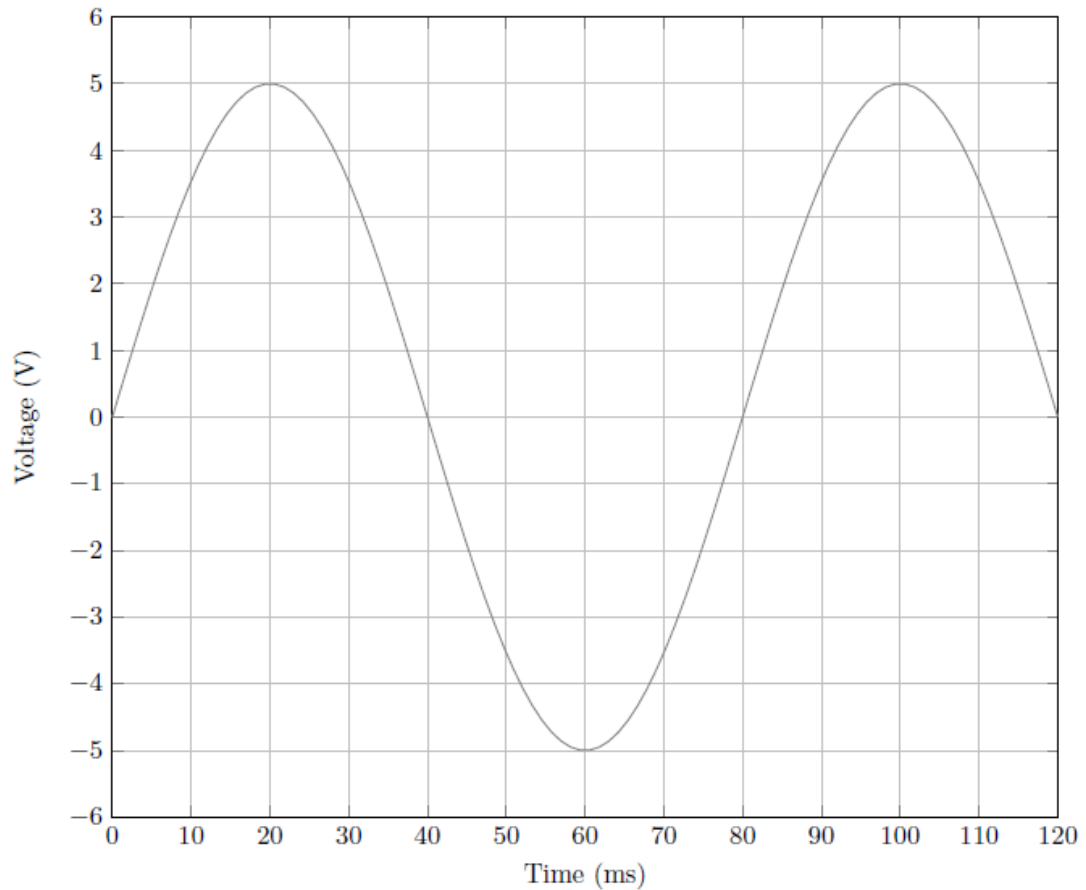
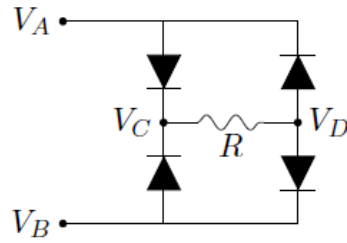
the current I is given by

$$I = I_0 \exp\left(\frac{-eV_0}{kT}\right) \left[\exp\left(\frac{eV}{kT}\right) - 1 \right],$$

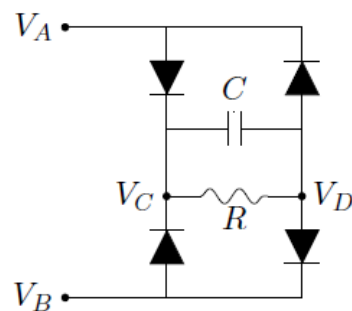
where $I_0 = 25 \mu\text{A}$ and $V_0 = 1.0 \text{ V}$, e is the elementary charge, k is the Boltzmann constant, T is the absolute (Kelvin) temperature, and $V = V_L - V_R$ is the potential difference (positive V when $V_L > V_R$ corresponds to current flowing from left to right).

Throughout this problem, assume low temperature, i.e. $kT \ll eV_0$.

- (a) Show some working and sketch a graph of I/I_0 against V/V_0 . [2]
- (b) In the circuit below, a sinusoidal input voltage is applied across $V_{AB} = V_A - V_B$ as shown in the graph. The resistance $R = 5.0 \, \Omega$.



- i. Show some working and sketch $V_{CD} = V_C - V_D$ for the same time interval as shown for V_{AB} . Assume that V_{AB} has been running for a long time. [3]
- ii. A capacitor $C = 50 \, \text{mF}$ is now added to the circuit, as shown below. [3]

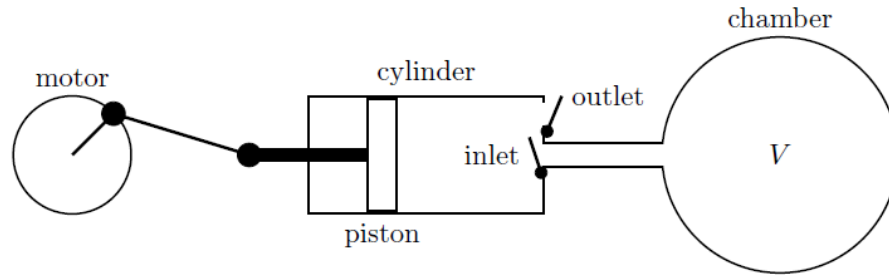


Assume that the same sinusoidal input voltage V_{AB} is applied as shown in the

graph, and has been running for a long time. Show some working and sketch the graph for V_{CD} as a function of time with the capacitor added.

Q6 total: 8

7. A vacuum system consists of a chamber of constant volume V connected to a pump mechanism in the form of a cylinder with a piston that moves left and right. As the piston moves, the minimum volume in the pump cylinder (to the right of the piston) is V_0 , and the maximum volume is $V_0 + \Delta V$. Assume that $\Delta V \ll V$.



The cylinder has two valves. The inlet valve opens when the pressure inside the cylinder is lower than the pressure in the chamber, but closes when the piston moves to the right. The outlet valve opens when the pressure inside the cylinder is greater than atmospheric pressure P_a , and closes when the piston moves to the left.

A motor drives the oscillatory motion of the piston. Each such complete pumping cycle takes a short time Δt . The piston moves at such a rate that heat is not conducted in or out of the gas contained in the cylinder during the pumping cycle. Assume that Δt is a very small quantity, but that $\Delta V/\Delta t \equiv \alpha$ is finite.

The gas in the chamber is ideal monatomic and remains at a fixed temperature of T_a . At time $t = 0$, the pressure inside the chamber is P_a . Start with the assumption that $V_0 = 0$ with the piston all the way to the right. Assume that there are no leaks in the system.

- (a) State, for an ideal monatomic gas, the value of the adiabatic gas constant [1]

$$\gamma = \frac{C_p}{C_v} ,$$

where C_p is the heat capacity at constant pressure and C_v is the heat capacity at constant volume.

- (b) Determine an expression for the chamber pressure $P(t)$ at a later time t , and show that in the limit where $\Delta V/V$ vanishes, the pressure can be written as [3]

$$P(t) = P_a \exp(-t/\tau_1) ,$$

where τ_1 is expressed in terms of variables introduced in the problem statement.

Hint: You may use the following mathematical definition of Euler's number,

$$e = \lim_{x \rightarrow 0} (1 + x)^{1/x}$$

- (c) Determine an expression for the temperature $T_{\text{out}}(t)$ of the gas as it is emitted from the pump cylinder into the atmosphere, and show that it can be written as [4]

$$T_{\text{out}}(t) = T_a \exp(t/\tau_2) ,$$

where τ_2 is expressed in terms of variables introduced in the problem statement.

- (d) Now assume that $0 < V_0 < \Delta V \ll V$. Determine an expression for the minimum possible pressure P_{\min} that is achievable in the chamber. You may express your answer in terms of P_a , V_0 and ΔV . [3]

Q7 total: 11

8. A photon with wavelength λ scatters at an angle θ off an electron of mass m initially at rest, as shown in Fig. 1. Denote the wavelength of the scattered photon as λ' .

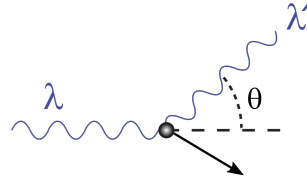


Figure 1: By JabberWok, CC BY-SA 3.0

<https://commons.wikimedia.org/w/index.php?curid=2078004>

- (a) Write down a relativistic expression for the electron energy E after the scattering event, in terms of its rest-mass m , the magnitude P of its momentum, and the speed of light in vacuum c . [1]
- (b) By considering momentum conservation, show that [3]

$$P^2 = h^2 \left(\frac{1}{\lambda^2} + \frac{1}{(\lambda')^2} - \frac{2 \cos \theta}{\lambda \lambda'} \right),$$

where h is the Planck constant.

- (c) By also considering energy conservation, show that [4]

$$\lambda' - \lambda = \lambda_C (1 - \cos \theta),$$

where λ_C is known as the Compton wavelength. Determine an expression for λ_C in terms of fundamental constants.

- (d) Compton's original experiment was based on X-rays bombarding a graphite target. He found that some X-rays experienced no wavelength shift despite being scattered through large angles. Suggest an explanation for this. [1]

Q8 total: 9

9. Quantum particles of integer spin are known as bosons. At low temperatures, a macroscopic number of bosons occupy the lowest energy quantum state, resulting in a collective quantum phase known as a Bose-Einstein condensate (BEC). The experimental achievement of BEC was honoured by the 2001 Nobel Prize in Physics.

In this question, we will estimate the critical temperature for BEC based on comparability of the de Broglie wavelength and the particle separation.

- (a) By considering the average kinetic energy of translational motion, determine an expression for the typical de Broglie wavelength λ of gas particles of mass m at temperature T . Use the symbol h for the Planck constant and k for the Boltzmann constant. [4]
- (b) Determine the typical linear separation d of gas particles as a function of mass density ρ . You may use symbols introduced in the previous part. [3]

- (c) Hence determine an expression for the critical temperature T_c for Bose-Einstein condensation. You may use symbols introduced in the previous parts. [2]
- (d) For a gas of Rubidium-87 atoms, a typical BEC temperature is $T_c = 100$ nK. For such a gas, calculate a numerical value for the ratio ρ_c/ρ_0 , where ρ_c is the corresponding mass density for BEC and ρ_0 is the density for a classical ideal gas at standard temperature and pressure $T_0 = 300$ K, $p_0 = 10^5$ Pa. [3]

Q9 total: 12

2018 CODATA adjustment

From: <http://physics.nist.gov/constants>**Fundamental Physical Constants — Frequently used constants**

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c	299 792 458	m s^{-1}	exact
Newtonian constant of gravitation	G	$6.674\,30(15) \times 10^{-11}$	$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$	2.2×10^{-5}
Planck constant*	h	$6.626\,070\,15 \times 10^{-34}$	J Hz^{-1}	exact
	\hbar	$1.054\,571\,817 \dots \times 10^{-34}$	J s	exact
elementary charge	e	$1.602\,176\,634 \times 10^{-19}$	C	exact
vacuum magnetic permeability $4\pi\alpha\hbar/e^2c$	μ_0	$1.256\,637\,062\,12(19) \times 10^{-6}$	N A^{-2}	1.5×10^{-10}
vacuum electric permittivity $1/\mu_0c^2$	ϵ_0	$8.854\,187\,8128(13) \times 10^{-12}$	F m^{-1}	1.5×10^{-10}
Josephson constant $2e/h$	K_J	$483\,597.848\,4 \dots \times 10^9$	Hz V^{-1}	exact
von Klitzing constant $\mu_0c/2\alpha = 2\pi\hbar/e^2$	R_K	$25\,812.807\,45 \dots$	Ω	exact
magnetic flux quantum $2\pi\hbar/(2e)$	Φ_0	$2.067\,833\,848 \dots \times 10^{-15}$	Wb	exact
conductance quantum $2e^2/2\pi\hbar$	G_0	$7.748\,091\,729 \dots \times 10^{-5}$	S	exact
electron mass	m_e	$9.109\,383\,7015(28) \times 10^{-31}$	kg	3.0×10^{-10}
proton mass	m_p	$1.672\,621\,923\,69(51) \times 10^{-27}$	kg	3.1×10^{-10}
proton-electron mass ratio	m_p/m_e	$1836.152\,673\,43(11)$		6.0×10^{-11}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5693(11) \times 10^{-3}$		1.5×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,084(21)$		1.5×10^{-10}
Rydberg frequency $\alpha^2m_e c^2/2h$	cR_∞	$3.289\,841\,960\,2508(64) \times 10^{15}$	Hz	1.9×10^{-12}
Boltzmann constant	k	$1.380\,649 \times 10^{-23}$	J K^{-1}	exact
Avogadro constant	N_A	$6.022\,140\,76 \times 10^{23}$	mol^{-1}	exact
molar gas constant $N_A k$	R	$8.314\,462\,618 \dots$	$\text{J mol}^{-1} \text{K}^{-1}$	exact
Faraday constant $N_A e$	F	$96\,485.332\,12 \dots$	C mol^{-1}	exact
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3c^2$	σ	$5.670\,374\,419 \dots \times 10^{-8}$	$\text{W m}^{-2} \text{K}^{-4}$	exact
Non-SI units accepted for use with the SI				
electron volt (e/C) J	eV	$1.602\,176\,634 \times 10^{-19}$	J	exact
(unified) atomic mass unit $\frac{1}{12}m(^{12}\text{C})$	u	$1.660\,539\,066\,60(50) \times 10^{-27}$	kg	3.0×10^{-10}

* The energy of a photon with frequency ν expressed in unit Hz is $E = h\nu$ in J. Unitary time evolution of the state of this photon is given by $\exp(-iEt/\hbar)|\varphi\rangle$, where $|\varphi\rangle$ is the photon state at time $t = 0$ and time is expressed in unit s. The ratio Et/\hbar is a phase.