Singapore Physics
Olympiad Training

2020 selection test for Asian and International Physics Olympiad travelling teams

1. This is a **four-hour** test. Attempt all questions. The maximum total score is **88**; marks allocated for each question part are indicated in square brackets.
2. Check that there are a total of **11 printed pages** (*including* this cover page). The last page contains a table of physical constants that you may refer to and use.
3. Begin your answer for each question on a **fresh sheet of paper**, and present all answers clearly. Your answer sheets should be properly sorted in order of the questions.
4. Write your name on the **top right hand corner of every answer sheet** you submit.
5. Please **complete and sign the declaration on page 2**, which should be **stapled together and submitted** with your answer sheets.
6. You may use a standard (non-programmable) scientific **calculator** in accordance with the statutes of the International Physics Olympiad.
7. No books or documents relevant to the test may be brought into the examination room.

Declaration

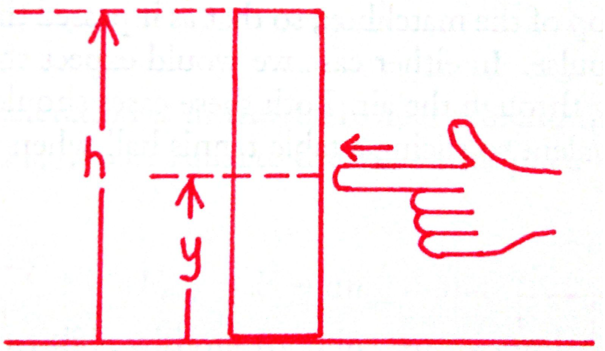
I declare that I will be fully committed to the training for and participation in the Asian Physics Olympiad and/or the International Physics Olympiad if selected for the travelling team. I will check first with the MOE coordinator before taking on additional commitments not listed below.

Potential limitations to my commitment in the period from now to end-July 2020 are described **exhaustively** in the box below, such as other academic competitions, CCA commitments (school-related or otherwise), travel plans, etc.

Name and signature: ________________________

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	5	6	5	5	11	7	4	12	6	27	88
Score:											

1. A box has mass m, and is of uniform density. It has height h when it stands upright on a table. The coefficient of friction between the surfaces is μ, and the acceleration of free fall is g.



   (a) How would you gently push the box to get it moving without toppling? [1]

   (b) Determine, in terms of the symbols introduced, the position y above the table surface at which to apply a force on the box, such that it remains upright and has an acceleration a. [4]

Q1 total: 5

2. A cylindrical beaker of water set spinning about its central axis results in a parabolic cross-section for the water surface. Mathematically, the height h of the water surface (measured from the lowest point on the water surface) is related to the radial distance r from the axis by

$$h = kr^2$$

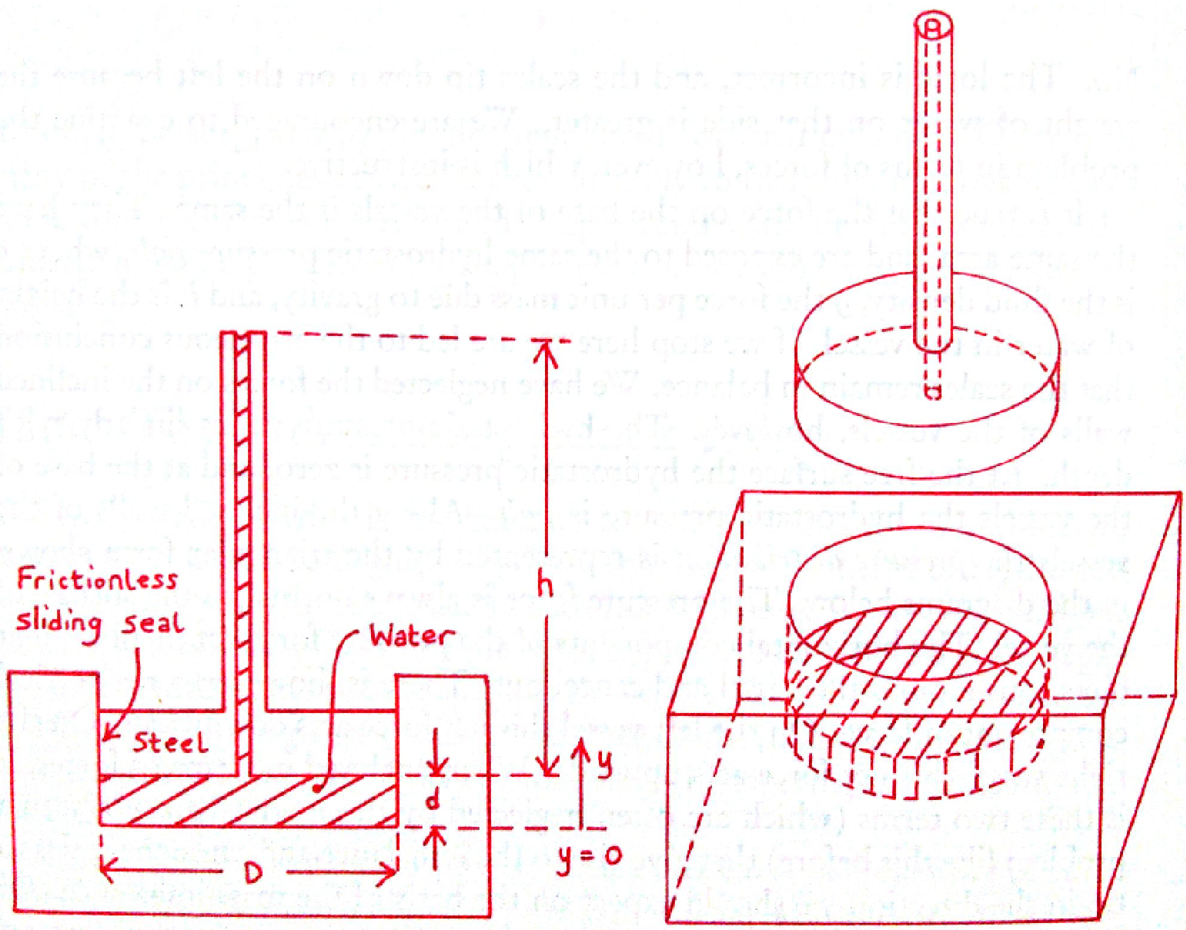
   (a) Prove this result and determine k in terms of the angular speed ω, the density of water ρ, and the acceleration of free fall g. [4]

   (b) Predict whether a small block of wood of density $\rho' < \rho$ placed on the sloping surface of the parabola will rise up or fall down the parabola. Explain your reasoning. [2]

Q2 total: 6

3. A thick cylindrical steel plate of diameter $D = 1$ m has a hole (of diameter $\delta = 1$ cm) drilled through the middle. A rigid narrow tube (of matching inner diameter δ), open at both ends, is attached and sealed to the top of the hole in the plate. The resulting column through the plate and tube has a height $h = 10$ m. The plate and tube form a piston assembly of mass $M = 10^3$ kg.

   The piston assembly is housed in a close-fitting cylindrical cavity, sealed with a frictionless seal that allows the piston to move up and down. The piston is initially held in a position such that the cavity underneath it is $d = 10$ cm deep. With the piston held in this position, water (of density $\rho = 10^3$ kg m^{-3}) is poured in slowly from the top of the tube, such that the entire volume of air is gradually and completely replaced by water. The tube is fully filled with water, and the system is released from rest.



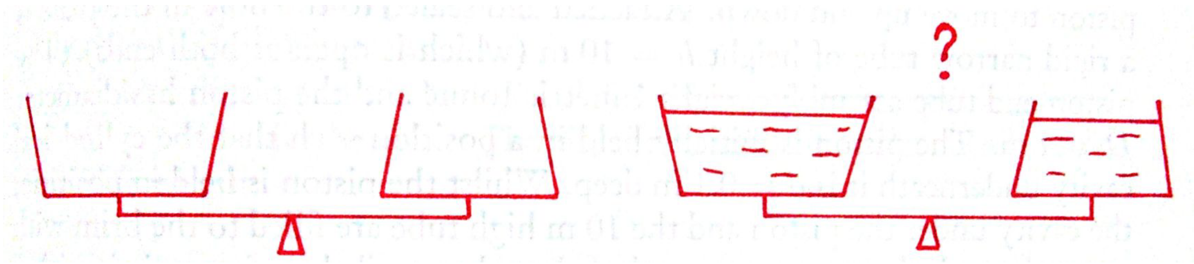
   (a) Calculate the equilibrium height of the water in the tube (measured from the bottom surface of the piston). [4]

   (b) Which direction did the piston move? [1]

Q3 total: 5

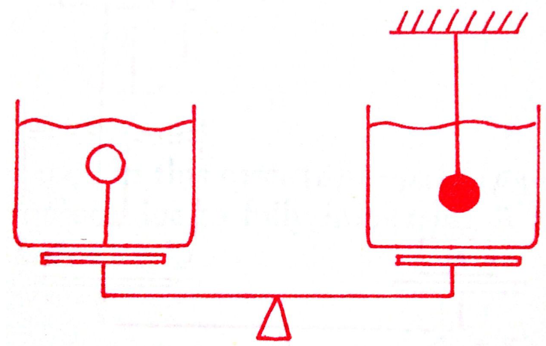
4. This question consists of a couple of puzzles involving weighing scales.

   (a) A pair of scales has integrated vessels (i.e. the vessels are part of the scales themselves rather than sitting on top of the scales) of the same base area. The left vessel tapers outwards to the top, while the right vessel tapers inwards to the top.



      i. Discuss the flaw(s) in the reasoning below: [2]
         "If the vessels are filled with water to the same height, the force on the base of the vessels should be the same and so the scales should remain balanced."

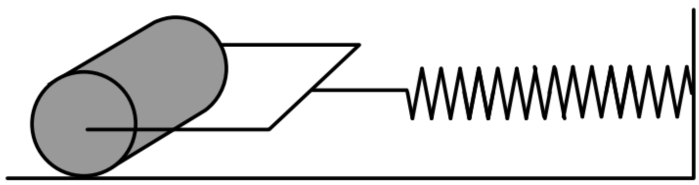
   (b) A pair of identical vessels contain identical amounts of water. In the left vessel, a very light ball is fully submerged and attached to the base by a very light string. In the right vessel, a very heavy ball (of the same volume as the other ball) is fully submerged and attached to the ceiling by a very light string.



      i. Do the scales tip left, tip right, or stay level? Explain. [3]

Q4 total: 5

5. A roller consists of a solid uniform cylinder of mass m and radius r. The roller is on a horizontal surface and is attached, by a ideal spring of spring constant k and negligible mass, to a vertical wall. The coefficient of friction between the roller and the horizontal surface is μ, and the acceleration of free fall is g.



   (a) If the roller performs small oscillations without slipping, show that the motion is simple harmonic and determine an expression for the period of oscillations T. [4]

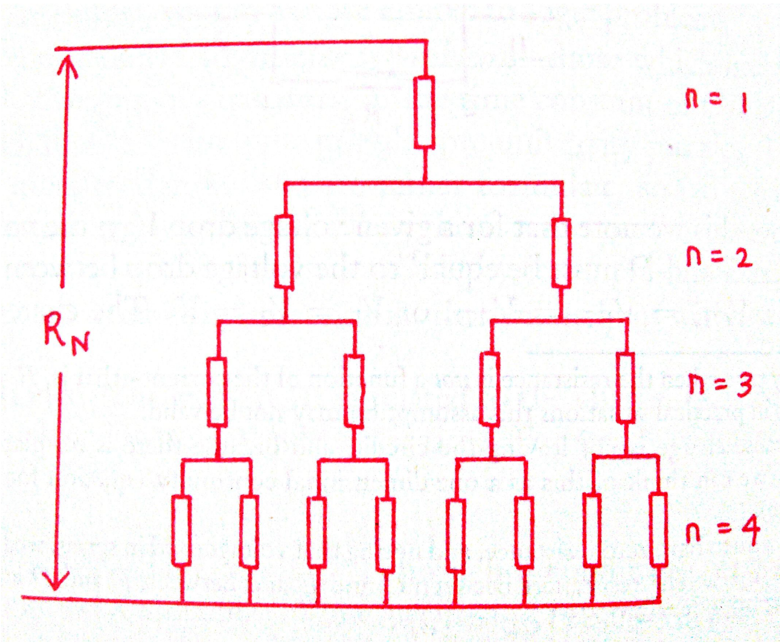
(b) Determine an expression for the maximum oscillation amplitude A_0 (measured by the deformation of the spring) such that the roller oscillates without slipping. [3]

(c) If the spring is stretched to an initial amplitude $A \gg A_0$, estimate the maximum angular speed ω of the roller after it is released. [4]

Q5 total: 11

6. This question involves resistor arrangements.

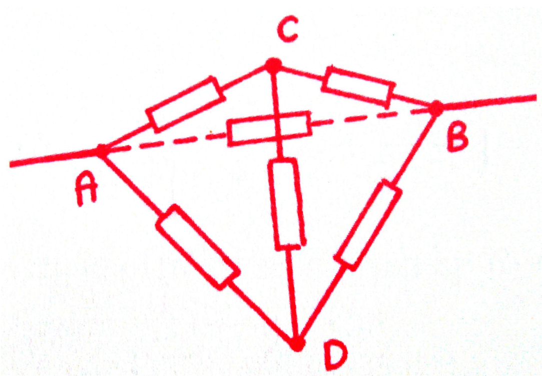
(a) A resistor pyramid is built in a branched structure as shown below.



i. If there are N levels, constructed from individual resistances of value R, what is the equivalent resistance R_N across the pyramid? [3]

ii. What is the limit R_∞? [1]

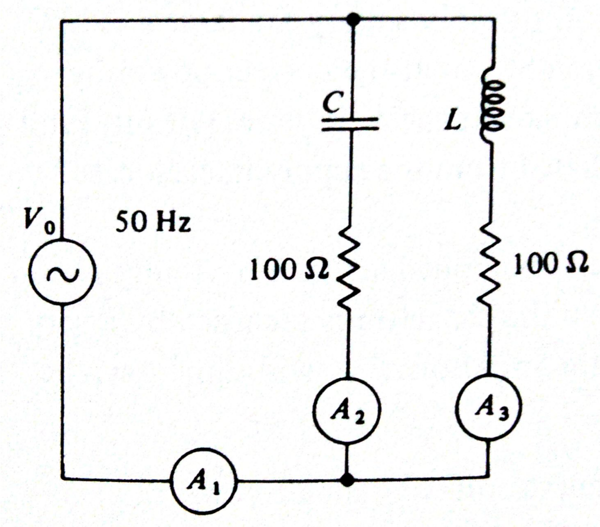
(b) A resistor tetrahedron is constructed from individual resistances of value R.



i. What is the equivalent resistance R_T across any pair of vertices (corners)? [3]

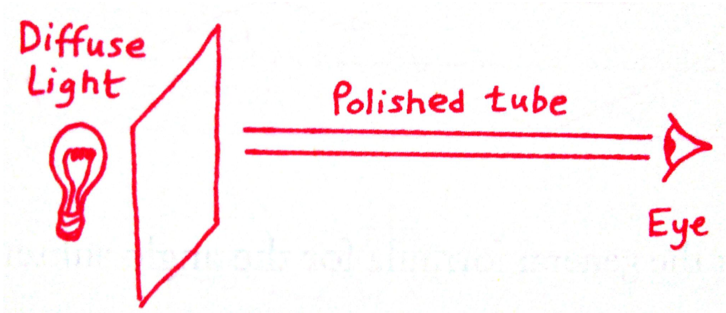
Q6 total: 7

7. The readings of the three a.c. ammeters are all equal. Calculate the values of L and C. [4]



Q7 total: 4

8. This question involves optical phenomena in two different contexts.
   (a) When you look with one eye at a diffuse source of light through a long, thin polished metal tube (e.g. a metre-long copper pipe used in domestic plumbing), you will see a pattern of rings.



At the centre there is a bright circle, which is the diffuse light seen directly through the hole. Surrounding this is a ring of light that is less bright than the central circle, and surrounding that is another ring that is less bright still, and so on. The rings become successively less bright until the tube appears dark.

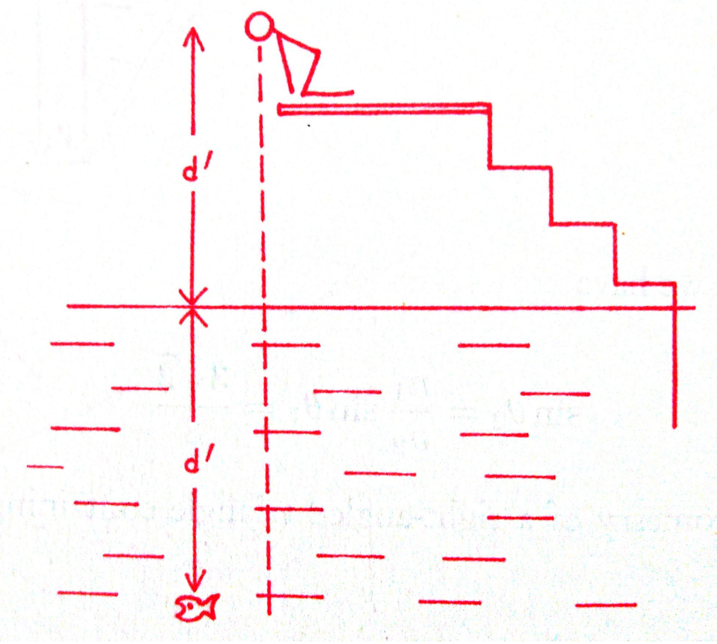
Denote the length of the tube as L and the inner diameter of the tube as D.

i. Derive an expression for the angle θ_n subtended at the eye by the n-th ring. [3]

ii. Show that the rings appear to have approximately the same width, and derive an expression for this width. [2]

(b) Binocular vision is useful for depth perception.

An observer, whose two eyes are a distance d' above the water surface, perceives a fish to be a distance d' directly (vertically) below the water surface. Let the separation between her eyes be $2s$. Let the refractive index of water be n and assume that the refractive index of air is 1.

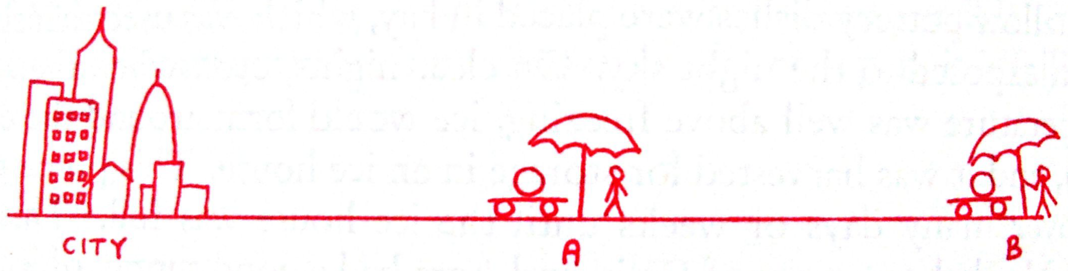


i. Assuming that $s \ll d'$, show that the true depth d of the fish below the surface is approximately given by $d = nd'$. [3]

ii. Now set $n = 4/3$ and derive an exact expression for d, *without* making the assumption that $s \ll d'$. [4]

Q8 total: 12

9. Assume a uniform desert outside the city. Entrepreneural ice resellers operate at point A and point B in the desert. The distance from the city to point B is double the distance from the city to point A, and it takes twice as long to travel from city to point B than it takes to go from city to point A.



The ice resellers can each buy a single solid uniform sphere of ice in the city. They then travel through the desert to their respective points A and B, journeying in the night so we can ignore radiation from the sun. Assume that the temperature of the desert is the same and constant during both journeys (this is the uniform desert approximation).

(a) If the reseller at point A leaves the city with a 800 kg sphere of ice and arrives at his stall with a 100 kg sphere of ice, what spherical mass of ice would the reseller at point B need to buy at the city in order to arrive at her stall with a 100 kg sphere of ice? [5]

(b) If the reseller at point A with his 100 kg sphere had continued on without stopping, directly towards point B (and at the same speed), would the ice have completely melted before reaching B, exactly at B, or after passing B? [1]

Q9 total: 6

10. In this question, we consider a toy model for a star, as a self-gravitating mass of fluid. We will assume that there are **no** nuclear or chemical reactions such that only mechanical (including gravitational) forces are at play. For simplicity, we will completely neglect relativistic effects.

Assume that the star has a sharp boundary, with radius R and spherical volume $\frac{4}{3}\pi R^3$.

Due to the spherical symmetry, the following are functions of radius r, and are non-negative for $0 < r \leq R$:

- the pressure $P = P(r)$,
- the (mass) density $\rho = \rho(r)$, and
- the number of atoms per unit volume, i.e., number density $n = n(r)$.

In addition, define $M = M(r)$ as the mass of fluid contained **within** a sphere of radius r.

(a) In this first part, we will focus more on mass and gravity.
Use the symbol G for the Newtonian gravitational constant.

i. Determine an expression for the mass gradient dM/dr in terms of ρ and r. [1]

ii. By considering an infinitesimal volume of fluid at radius r, determine an expression for the pressure gradient dP/dr in terms of the symbols introduced. [3]

iii. Hence show that the central pressure P_0 is given by [3]

$$P_0 = \frac{G}{4\pi}\int_0^{M_T} \frac{M}{r^4} dM \ ,$$

where M_T is the total mass of the star.

iv. Show that a crude lower bound is given by [1]

$$P_0 > \frac{GM_T^2}{8\pi R^4} \ .$$

v. By integrating the enclosed volume $V = \frac{4}{3}\pi r^3$ over the pressure from the centre of the sphere to the boundary of the sphere, show that this integral is negative and has a value equal to one-third of the gravitational potential energy Ω contained in the star, i.e., [3]

$$\int V\, dP = \frac{\Omega}{3} \ .$$

(b) In this second part, we will introduce the additional assumption that the fluid behaves like a classical (non-relativistic) ideal gas.

While this might seem like a crazy assumption to make given the pressures and temperatures involved, this might still be plausible if we imagine that matter exists as a plasma of nuclei and electrons. Since atomic nuclei are orders of magnitude smaller than the electron clouds of atoms, the "ideal gas" assumptions can still hold at high particle number densities. Nonetheless, we will *ignore* the electromagnetic energy of the plasma in this question.

Let c_v be the constant-volume (isochoric) heat capacity per particle of gas and c_p be the constant-pressure (isobaric) heat capacity per particle of gas. Define

$$\gamma = \frac{c_p}{c_v}$$

as the ratio of the isobaric and isochoric heat capacities.

i. Using the first law of thermodynamics, show that $c_p = c_v + k$ for an ideal gas, where k is Boltzmann's constant. [2]

ii. Show that the internal (thermal) energy density u, which is the internal (thermal) energy per volume of gas, is proportional to the pressure P and given by [3]

$$u = \frac{P}{\gamma - 1} \ .$$

iii. Hence show that the internal (thermal) energy U of the star is related to its gravitational potential energy through what is known as the virial theorem, [2]

$$\Omega = -3(\gamma - 1)U \ .$$

(c) In this final part, we will consider the physics of this toy model for different values of γ.

i. Show that $\gamma = 5/3$ for a "monatomic" ideal gas. [1]

ii. Show that the total energy of the star is negative for such a gas. [1]

iii. If we now consider that such a star is constantly radiating energy in the form of photons, suggest whether the star is expected to get smaller or bigger, and whether the temperature is expected to increase or decrease with time. [2]

iv. Assume that our Sun is such an ideal gas composed of an equal mixture of protons and electrons (i.e., fully ionised hydrogen atoms). Estimate the mean temperature of the Sun, using the following data for the Sun: $\Omega \sim -4 \times 10^{41}$ J; $M_T \sim 2 \times 10^{30}$ kg. [3]

v. Determine the value of γ such that the total energy of the star is zero. [1]

vi. Discuss the stability of the stellar structure in this case. [1]

Q10 total: 27

Physical constants

Speed of light in vacuum	c	=	$299\,792\,458 \text{ m} \cdot \text{s}^{-1}$
Vacuum permeability (magnetic constant)	μ_0	=	$4\pi \times 10^{-7} \text{ kg} \cdot \text{m} \cdot \text{A}^{-2} \cdot \text{s}^{-2}$
Vacuum permittivity (electrical constant)	ε_0	=	$8.854\,187\,817 \times 10^{-12} \text{ A}^2 \cdot \text{s}^4 \cdot \text{kg}^{-1} \cdot \text{m}^{-3}$
Elementary charge	e	=	$1.602\,176\,620\,8(98) \times 10^{-19} \text{ A} \cdot \text{s}$
Mass of the electron	m_e	=	$9.109\,383\,56(11) \times 10^{-31} \text{ kg}$
		=	$0.510\,998\,946\,1(31) \frac{\text{MeV}}{\text{c}^2}$
Mass of the proton	m_p	=	$1.672\,621\,898(21) \times 10^{-27} \text{ kg}$
		=	$938.272\,081\,3(58) \frac{\text{MeV}}{\text{c}^2}$
Mass of the neutron	m_n	=	$1.674\,927\,471(21) \times 10^{-27} \text{ kg}$
		=	$939.565\,413\,3(58) \frac{\text{MeV}}{\text{c}^2}$
Unified atomic mass unit	u	=	$1.660\,539\,040(20) \times 10^{-27} \text{ kg}$
Rydberg constant	R_∞	=	$10\,973\,731.568\,508(65) \text{ m}^{-1}$
Universal constant of gravitation	G	=	$6.674\,08(31) \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$
Acceleration due to gravity (in Zurich)	g	=	$9.81 \text{ m} \cdot \text{s}^{-2}$
Planck's constant	h	=	$6.626\,070\,040\,(81) \times 10^{-34} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-1}$
Avogadro number	N_A	=	$6.022\,140\,857\,(74) \times 10^{23} \text{ mol}^{-1}$
Molar gas constant	R	=	$8.314\,4598(48) \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{mol}^{-1} \cdot \text{K}^{-1}$
Molar mass constant	M_u	=	$1 \times 10^{-3} \text{ kg} \cdot \text{mol}^{-1}$
Boltzmann constant	k_B	=	$1.380\,648\,52(79) \times 10^{-23} \text{ kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \cdot \text{K}^{-1}$
Stefan-Boltzmann constant	σ	=	$5.670\,367\,(13) \times 10^{-8} \text{ kg} \cdot \text{s}^{-3} \cdot \text{K}^{-4}$