Singapore Physics Olympiad Training

2019 selection test for Asian and International Physics Olympiad travelling teams

1. This is a **four-hour** test. Attempt all questions, the maximum total score is **80**. Marks allocated for each part of a question are indicated.
2. Check that there are a total of **8 printed pages** (including this cover page). The last page contains a table of physical constants that you may refer to.
3. Begin your answer for each question on a **fresh sheet of paper**, and present all answers clearly.
4. Write your name on the **top right hand corner of every answer sheet** you submit.
5. This cover sheet should be **stapled together** with your answer sheets, and the answer sheets should be properly sorted.
6. No books or documents relevant to the test may be brought into the examination room.
7. Please **complete and sign the declaration** on page 2.

Declaration

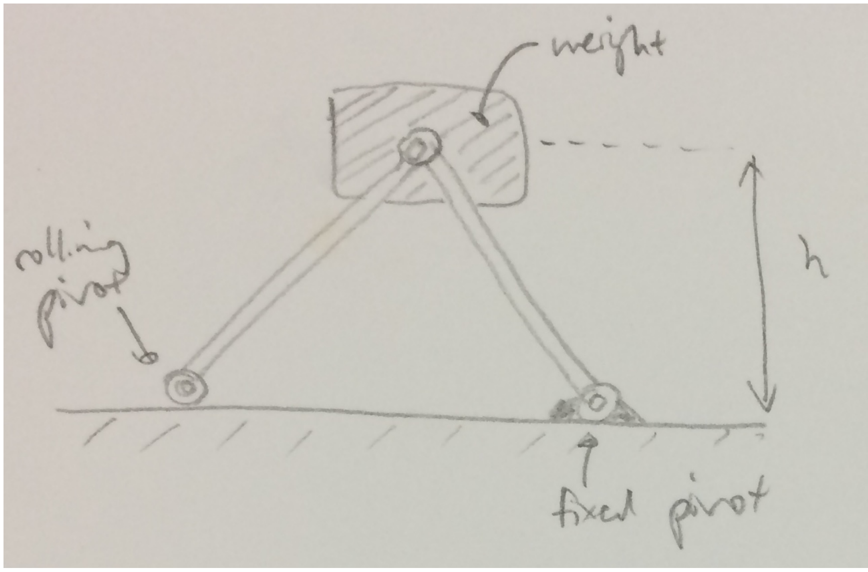
I declare that I will be fully committed to the training for and participation in the Asian Physics Olympiad and/or the International Physics Olympiad if selected for the travelling team. I will check first with the MOE coordinator before taking on additional commitments not listed below.

Potential limitations to my commitment in the period from now to end-July 2019 are described **exhaustively** in the box below, such as other academic competitions, CCA commitments (school-related or otherwise), travel plans, etc.

Name and signature: ______________________________

Question:	1	2	3	4	5	6	Total
Points:	12	16	15	14	10	13	80
Score:							

1. A well-oiled machine of some kind (see figure) has a weight (of mass m) attached with a pivot through its centre of mass (at height h above the ground) to two rigid rods (of equal length L) with negligible mass.



The rods are pivoted (like an elbow joint) at both ends. One rod has its bottom end pivoted to a fixed pivot on the floor, while the other rod has its bottom end pivoted to a rolling pivot that can move along a straight horizontal track.

Denote the local gravitational field strength as g.

(a) Find the horizontal force F that has to be applied at the rolling pivot to keep the machine in equilibrium. [Express your answer as a function of the variables introduced.] [2]

(b) Suppose now that the rolling pivot is moving at constant speed u towards the fixed pivot.

i. State the horizontal speed of the weight. [1]

ii. State the direction of the acceleration a of the weight, and briefly explain why the horizontal force F that now has to be applied at the rolling pivot is different from your answer in (a). [2]

iii. By considering the circular trajectory of the weight, find its tangential speed v. [2]

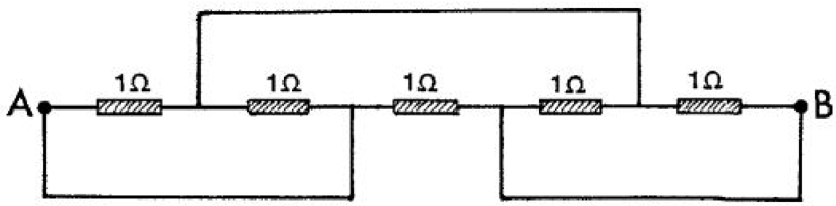
iv. Find the radial acceleration a_r of the weight, taking the origin as the fixed pivot, and hence find the acceleration a of the weight. [2]

v. Find the resultant force on the weight, and hence find the horizontal force F that now has to be applied at the rolling pivot. [3]

Q1 total: 12

2. This question consists of short puzzles, which can be solved independently.

(a) Five 1 Ω resistors are connected in a circuit as shown in the figure. Determine the resulting resistance R between points A and B. [3]



(b) A thermally insulated piece of metal is heated by an electric current at a constant power P. This results in the absolute temperature T of the metal varying with time t as follows: [3]

$$T(t) = T_0[1 + a(t - t_0)]^{1/4} ,$$

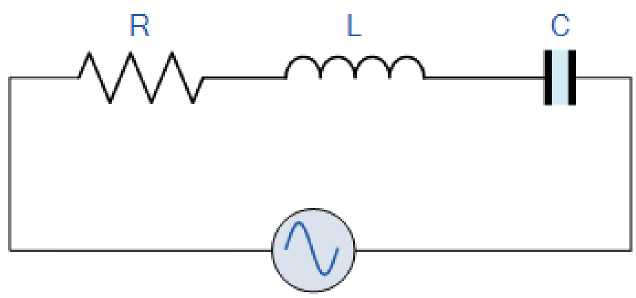
where T_0, a, and t_0 are constants.

Determine the heat capacity $C(T)$ of the metal for the temperature range of this experiment.

(c) An electron was accelerated from rest through a voltage of 1 MV. What is its speed v and momentum p? [3]

(d) A neutral pion is travelling with speed v when it decays into two photons, which are seen to emerge at equal angles θ on either side of the original velocity. Show that $v = c\cos\theta$. [3]

(e) In the circuit below, the input voltage is $V_{in} = V_0 \sin(\omega t)$, where V_0 and ω are constants (peak voltage and angular frequency respectively), and t is the time. [4]



Find an expression for the peak current I_0 in terms of R, L, C, V_0, and ω.

Also comment on how your answer depends on the value of ω.

Q2 total: 16

3. This question asks for some predictions, which can be tackled independently.

(a) A small mass hangs on the end of a massless ideal spring and oscillates up and down at its natural frequency f_0. If the spring is cut in half and the mass reattached at the end, what is the new frequency? [3]

(b) A type of helicopter can hover if the mechanical power output of its engine is P_0. What is the mechanical power output required for an exact 1.5-scale replica (i.e., the replica is 50% larger in all linear dimensions) to also hover? [4]

(c) On a given day, the air is dry and has a density 1.2500 kg/m^3. The next day, the humidity has increased such that the air consists of 2% by mass of water vapour. If the pressure and temperature are the same as the day before, what is the air density? [4]

[**Data:** mean molar mass of dry air is 28.8 g/mol, molar mass of water 18 g/mol.]

(d) The mean temperature of the earth is 287 K. What would the new mean temperature be if the mean distance between the Earth and the Sun was reduced by 1%? [4]

Q3 total: 15

4. In a semiconductor, the average (drift) velocity of the charge carriers in the presence of an electric field only is

$$v = \mu E \ ,$$

where μ is called the mobility. If a magnetic field is also present, the electric field is no longer parallel to the current. This phenomenon is known as the Hall effect.

Consider a long bar made from the semiconductor InSb, whose current carriers are electrons. The bar has the shape of a rectangular parallelepiped with sides a, b, and c (such that $a \gg b \gg c$). The bar is in an external magnetic field B which is parallel to the edge c, and the electric current I in the bar is parallel to the edge a.

The magnetic field produced by the current I can be neglected.

[**Data:** The electron mobility in InSb is $\mu = 7.8\ \mathrm{m^2/(V\,s)}$, the electron concentration in InSb is $n = 2.5 \times 10^{22}\ \mathrm{m^{-3}}$, the current is $I = 1.0$ A, the magnetic field is $B = 0.10$ T, and the dimensions of the bar are $a = 10$ cm, $b = 1.0$ cm, and $c = 1.0$ mm. The charge of an electron is $q = -e$.]

(a) Calculate the average (drift) velocity of an electron in the material. [2]

(b) Determine the magnitude and direction of the electric field in the bar. [5]

(c) Calculate the electric potential difference between opposite points on the surfaces of the bar in the direction of the edge b. [2]

(d) Find the analytic expression for the DC component of the electric potential difference in (c) if the current and the magnetic field are alternating; $I = I_0 \sin \omega t$ and $B = B_0 \sin(\omega t + \delta)$. [Express your answers in terms of the symbols given.] [5]

Q4 total: 14

5. A circular ring of thin copper wire is set rotating rapidly about a vertical diameter at a point within the Earth's magnetic field. The magnetic flux density of the Earths magnetic field at this point is 44.5 μT directed at an angle of 64° below the horizontal. [10]

[**Data:** Density of copper is $8.90 \times 10^3\ \mathrm{kg\,m^{-3}}$ and its resistivity is $1.70 \times 10^{-8}\ \mathrm{W\,m}$.]

Calculate how long it will take for the angular velocity of the ring to halve.

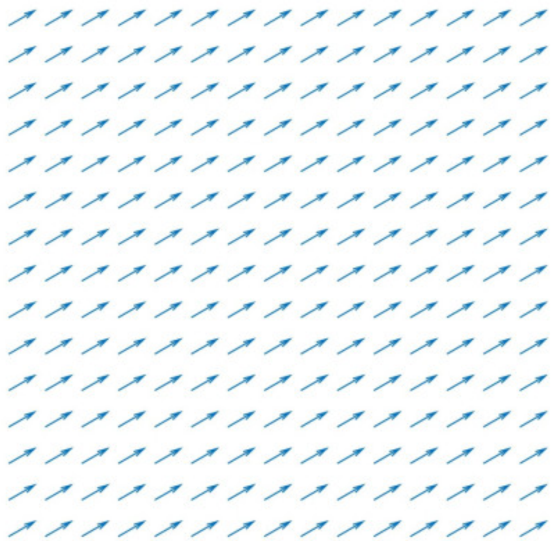
[You may assume that this time is much longer than the time for one revolution, that the frictional effects of the supports and air are negligible, and for the purposes of this question you should ignore self-inductance effects, although these would not be negligible.]

Q5 total: 10

6. The Berezinskii-Kosterlitz-Thouless transition (BKT transition) is an example of a topological phase transition, named after condensed matter physicists Vadim Berezinskii, John M. Kosterlitz and David J. Thouless. BKT transitions can be found in several 2-D systems in condensed matter physics, including Josephson junction arrays and thin disordered superconducting granular films. Work on the transition led to the 2016 Nobel Prize in Physics being awarded to Thouless, Kosterlitz and Duncan Haldane.

In this problem, we study a simplified model of the BKT transition to gain a flavour of the physics involved. In this simplified model, imagine we have a solid layer of particles (i.e. a thin film). The particles are fixed in position on a square grid with unit spacing between nearest neighbours. Each particle has a "direction arrow" of fixed length that is constrained to point within the layer. We call this arrow the classical spin vector.

The ground state (lowest energy configuration) occurs when the classical spin vectors are all aligned, shown schematically in the figure below.



The square grid has L particles in each row going from left to right (along the x-axis) and L particles in each column going from bottom to top (along the y-axis).

Let $\theta(x, y)$ be the angle (relative to the positive x-axis) indicating the direction in which the classical spin vector points for the particle at the coordinate position (x, y).

Define a function that maps this angle to a planar vector,

$$\vec{f}(\theta) \equiv \left(\frac{d\theta}{dx}\right)\hat{x} + \left(\frac{d\theta}{dy}\right)\hat{y} \; ,$$

where $\hat{x}, \hat{y}$ are unit vectors in the positive x-direction and positive y-direction respectively.

[Note: we're not being that careful with the mathematics here in this question, by using calculus to conveniently describe what is actually a system with a finite-size grid spacing.]

The energy of the system is given by the integral

$$E = \frac{J}{2} \int \int dx \, dy \, |\vec{f}|^2$$

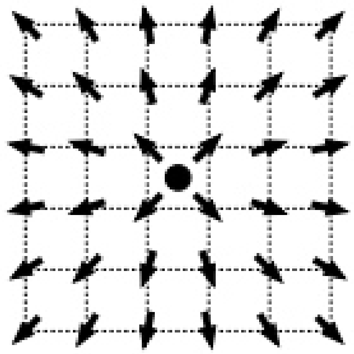
over the entire area of the system, where J is a constant with dimensions of energy. Note that $E = 0$ for a ground state.

(a) Suppose that we have an arbitrary arrangement of classical spin vectors. By considering a rectangular "closed loop" whose sides are parallel to the x- and y-axes, show that the integral of $\vec{f}$ along the closed loop is quantised, and write down the allowed values of [2]

$$Q = \oint \vec{f} \cdot d\vec{l} \; ,$$

where $d\vec{l}$ is the infinitesimal length element along the closed loop. We call Q the topological charge.

(b) Now consider a special pattern of classical spins shown in the figure below.



Taking the central "dot" as the origin, this pattern is described by

$$\theta(x, y) = \tan^{-1}\left(\frac{y}{x}\right) .$$

i. Find an expression for $\vec{f}$ in terms of x and y. [2]
[**Hint:** $\frac{d}{dx}\left(\tan^{-1} x\right) = 1/(1 + x^2)$.]

ii. Find the value of Q for this spin arrangement. [1]

iii. Sketch another pattern of classical spins (that looks qualitatively different!) with the same value of Q. [1]

iv. By making suitable approximations given that $L \gg 1$, find an (approximate) expression for E, and show that this is proportional to $\ln(L)$. [2]

v. Estimate the entropy S associated with the placement of this pattern of classical spins on the square grid. [2]

vi. Find an expression for the free energy $F = E - TS$, where T is the absolute temperature, and hence estimate the critical temperature T_c of the topological phase transition. [3]

Q6 total: 13

Physical constants

Speed of light in vacuum	c	=	$299\,792\,458\ \mathrm{m \cdot s^{-1}}$
Vacuum permeability (magnetic constant)	μ_0	=	$4\pi \times 10^{-7}\ \mathrm{kg \cdot m \cdot A^{-2} \cdot s^{-2}}$
Vacuum permittivity (electrical constant)	ε_0	=	$8.854\,187\,817 \times 10^{-12}\ \mathrm{A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}}$
Elementary charge	e	=	$1.602\,176\,620\,8(98) \times 10^{-19}\ \mathrm{A \cdot s}$
Mass of the electron	m_e	=	$9.109\,383\,56(11) \times 10^{-31}\ \mathrm{kg}$
		=	$0.510\,998\,946\,1(31)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Mass of the proton	m_p	=	$1.672\,621\,898(21) \times 10^{-27}\ \mathrm{kg}$
		=	$938.272\,081\,3(58)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Mass of the neutron	m_n	=	$1.674\,927\,471(21) \times 10^{-27}\ \mathrm{kg}$
		=	$939.565\,413\,3(58)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Unified atomic mass unit	u	=	$1.660\,539\,040(20) \times 10^{-27}\ \mathrm{kg}$
Rydberg constant	R_∞	=	$10\,973\,731.568\,508(65)\ \mathrm{m^{-1}}$
Universal constant of gravitation	G	=	$6.674\,08(31) \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$
Acceleration due to gravity (in Zurich)	g	=	$9.81\ \mathrm{m \cdot s^{-2}}$
Planck's constant	h	=	$6.626\,070\,040\ (81) \times 10^{-34}\ \mathrm{kg \cdot m^2 \cdot s^{-1}}$
Avogadro number	N_A	=	$6.022\,140\,857\ (74) \times 10^{23}\ \mathrm{mol^{-1}}$
Molar gas constant	R	=	$8.314\,4598(48)\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot mol^{-1} \cdot K^{-1}}$
Molar mass constant	M_u	=	$1 \times 10^{-3}\ \mathrm{kg \cdot mol^{-1}}$
Boltzmann constant	k_B	=	$1.380\,648\,52(79) \times 10^{-23}\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}}$
Stefan-Boltzmann constant	σ	=	$5.670\,367\ (13) \times 10^{-8}\ \mathrm{kg \cdot s^{-3} \cdot K^{-4}}$