Examiner's Report for 2017 selection test

Students should avoid asking for marks and focus on demonstrating that you know physics well through your work. Examiners are unlikely to be sympathetic and might in fact be harsher than usual.

If your handwriting is not very legible, increase the font size by a factor of 2 or so! Legibility is of critical importance.

Please also adhere to important instructions, such as beginning your answer to each question on a fresh sheet of paper.

Q1. Billiards

Several students had trouble with this question. For some, an elastic collision was assumed, when this was not necessary. One student didn't assume an elastic collision, but mentioned that the collision was assumed to be elastic!

Quite a few students struggled to solve it after drawing in all possible forces and getting overwhelmed by the complications. Important to take a step back and rethink the problem.

Some students seemed to not clearly understand what “normal contact” force means - for a sphere, “normal” must be along a radius.

Quite a few students used the formula given for a spherical shell without considering that the billiard ball was meant to be a solid sphere of uniform density.

Finally, there was an alarming lack of checks of reasonableness. The answer for the height cannot exceed the diameter of the ball, and probably cannot be less than the radius.

Q2. Oscillations

This question is quite similar in style to Olympiad questions.

(a)(i) This was well answered by most students.

(a)(ii) The easiest way to approach this is to compare the energy expression with the expression obtained for a simple harmonic oscillator. Nonetheless, it was interesting to see students produce good answers by deriving equations of motion from setting dE/dt to zero from conservation of mechanical energy.

(b)(i) Most students were able to see that the kinetic energy term was expressed simply in terms of ds/dt.

The way to use dimensional analysis is to first identify the variables that might be important to get the potential energy term. These would be the mass, the angular frequency, and some length scale. Only then do you figure out the combination that gives the appropriate units.

The more direct and physically insightful way to solve this part is to realise that the energy expression must be that for a simple harmonic oscillator.

(b)(ii) Prior knowledge was not necessarily useful in this question. Some students knew that the solution was a cycloid, but were not able to derive it. Others mistook the tautochrone for the brachistochrone and attempted to minimise the travel time, which is incorrect.

The physical insight needed is to link the potential energy mgy to the form of the expression proportional to s^2 from (b)(i), which links y and s. The rest is mathematics, which is a skill you need, and requires logical thinking as well.

Be clear on your objective. We have an equation involving y and s. Want to change to an equation involving y and theta. Now theta is linked to dy/dx, and also to infinitesimal ds by Pythagoras' theorem. So if you differentiate the equation with y and s, with respect to s, you can slowly replace all appearances of s in terms of y, and all appearances of ds in terms of dy and theta.

After the mathematics, you have y in terms of theta. Then you already know dy/dx and so you can integrate to get x in terms of theta too.

Mathematically, the parametric equations define a cycloid, which is the trajectory of a point on circumference of a circle as it rolls (without slipping) on a horizontal surface.

Q3. Soap Bubble

This question was generally well answered, though many made the mistake of taking 5 cm as the radius instead of the diameter as specified. Such mistakes cause unnecessary loss of marks.

Some students managed to explain that the charge distribution expected should be that of a conductor, since the bubble is mostly water.

There were some answers that led inexplicably to negative potentials, as though there were some mechanism for charge transfer during bubble bursting.

Q4. Balance

Some students managed to solve this efficiently. This question requires some familiarity with the concept of impedance. With an alternating voltage, the current through a resistor, capacitor, or inductor is proportional to the voltage, but with a possible phase difference.

The impedance formulas can always be quickly derived using complex numbers, by representing a sinusoidally varying voltage as V_0 exp(i omega t).

The zero signal condition for balance is that the ratio of voltages on both sides of each arm are equal, which, after dividing by current in each branch, leads to the corresponding ratio of impedances being equal. Solve for both the real part and the imaginary part of the equation and you get the solution.

Disappointingly, hardly anyone who solved it correctly commented on the interesting property that the solution was independent of the angular frequency of the voltage source.

Q5. Relativistic Doppler

This question was quite guided, and it was good to see that most students answered it very well.

(b)(ii) Most students could recall the Lorentz transformation equations appropriately.

(c)(i) Most students could recall the velocity addition formula appropriately.

(c)(ii) Most students could derive this well, by finding the time when the second pulse passes Alice and actually reaches Bob. Some managed to derive it by working in the frame of Alice, which also works of course, but requires transforming again to Bob's frame.

(c)(iii) Most could explain this, though some made careless errors in getting the expression.

(c)(iv) It was important to notice that u and c can take different roles in Eqn 5.1, as the limit described for this part gives an expression that applies for sound waves too. Students should not use the expression from (c)(iii) but the original expression Eqn 5.1.

Q6. Superfluid

This question appears novel, though it can be answered using classical physics. Olympiad questions are commonly posed in such a way to introduce advanced physics in a simplified fashion. Physics generally works this way as well - we might get a reasonable physical estimate using dimensional analysis, and get more precise numerical factors with increasingly sophisticated theories that agree also with experimental evidence.

(a) Relativistic expressions are not required, and are unnecessarily complicated as we are interested in low energy excitations in this question (but you might not know this, of course). It is a good idea to keep things simple though, unless being too simple is not good enough.

We can basically write two equations, corresponding to conservation of linear momentum and conservation of energy. The energy of the quasiparticle includes it's rest mass energy, if any.

(b) The expression to be shown gives a clue that we should eliminate v. Your equation has a term with the scalar product between u and p, and this is bounded. You should be able to argue that the equation cannot be satisfied if the inequality not true.

Some students attempted to use differentiation to argue about stationary points. A graphical approach is often more useful, though less precise, as it allows you to get a good feel and understanding of the mathematics.

(c)(i) Surprisingly few students answered this adequately. If the inequality in (b) was satisfied, then quasiparticles could be created, there are interactions, and the system is not in a superfluid phase.

So what is the criterion? If the minimum over p of E/p is non-zero, then the system is superfluid if the fluid particles travel with speed less than that minimum. Otherwise, the system is not superfluid.

(c)(ii) For 1, the minimum of E/p is zero, so the system is not superfluid. For 2, the minimum of E/p is c, which was meant as the speed of sound waves in the fluid, and so the system is superfluid if the particles are slower than that. For 3, the minimum of E/p can be obtained graphically by a tangent line to the curve, and the system is superfluid if the particles are slower than that value.

Q7. Cooling

There were students who did the first parts of this question well, but it appeared that many seemed to have run out of time and either rushed it or left many parts or even the whole question unanswered.

(a)(i) Many students were able to invoke the Boltzmann distribution, but explanations could generally be clearer and more specific. The Boltzmann distribution gives the exp(-E/kT) factor, and students should realise that for the 2D gas, the energy is purely kinetic (no interactions between particles and thus no potential energy) and given by mv^2/2.

A common mistake made by students was a blind application of formulas for 3D particles, e.g. using 3/2 kT for the thermal energy even though there are only two degree of freedom.

(a)(ii) The normalisation of probabilities was generally well done.

(a)(iii) Several students recognised that this was analogous to a transformation into polar coordinates. Here we are in velocity space rather than coordinate space, but the concept is identical.

(b)(i) Most of the students who reached this point were able to perform the integration, though some experienced mathematical difficulties.

(b)(ii) This was well answered.

(b)(iii) Fewer students were able to understand that the energy of the gas was purely kinetic energy and could be calculated as the expectation value of mv^2/2 over the probability density from (a)(iii).

There was also some confusion about whether to multiply the expectation value by N or N', though hindsight should yield the realisation that because the integral limits were already truncated, using N is the correct option.

(c)(i) Most students were able to explain this, though some answers were not accepted as it was important to not just say energy was lost, since particles were also lost. The concept of evaporative cooling from losing the more energetic particles was crucial to the conclusion that the average energy was reduced.

(c)(ii) Many students failed to account for the fact that only N' particles were left to thermalize at temperature T'. Here the fact that the gas had two degrees of freedom was important, so that the total energy from (b)(iii) was set to N'kT'.

(c)(ii) This was generally well done.

(d)(i) A common error was to omit the N'/N factor.

(d)(ii) This should be a curve that tends to infinity at zero eta and tends to 1 at infinite eta.

(d)(iii) The intended answer was not seen, as the concept of phase space density was probably not sufficiently developed in the question.

The increased phase space density suggests that repeated removal of high speed particles doesn't "thin" the gas too drastically, and very low temperatures can be achieved with evaporative cooling.

Q8. Time-of-flight Thermometry

Most students displayed adequate skills in working with data, graphing and analysing.

(a) Most managed to complete the dimensional analysis, though some tried to involve gravitational acceleration.

(b)(i) Most students answered poorly by arguing that the gravitational force is weak. Whether something is weak is comparative - the atoms are neutral and so electromagnetic forces are even weaker!

The key insight is to realise that gravity acts equally on all atoms, so the result is a bulk movement of the cloud and has no effect on the relative motion of the atoms in the cloud, so it does not increase or decrease the spreading of the cloud in the vertical direction. (But it does cause the entire cloud to accelerate downwards!)

(b)(ii) There was some confusion regarding whether to include a constant term in the expression. The constant term from (a) should not appear here, as the question asked for the difference between the mean squared radius at time t and at time 0.

It is possible to derive this more properly by considering ballistic motion of the atoms, but this was omitted to shorten the paper.

(c) Several students made mistakes with powers of ten, which was lamentable. It is very important to keep track of these.

A common conceptual mistake is thinking that plotting <R^2> against t^2 is the same as plotting ✓<R^2> against t. The problem comes when the value of <R^2> is non-zero at time 0, which is the case in this question. The linear trend for the former plot will be equivalent to a linear trend for the latter plot! This might seem like a subtle point, but is of great importance, and you should always write down the mathematics explicitly to make sure you get this straight.

-ZDT
27 March 2017