36th Singapore Physics Olympiad Theory Paper 2

Organised by

Institute of Physics

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Ministry of Education Singapore

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Instructions to Candidates

1. This is a 2-hour paper.
2. This paper consists of five (5) questions printed on twelve (12) pages. The second page is a General Information Sheet.
3. Attempt all questions.
4. Write your answers in the space provided in the question booklet.
5. You may request working paper from the invigilators.
6. You may not refer to any books or documents relevant to the competition.

NAME:____________________________________ INDEX NO: __________

SCHOOL: __________________________________

For Examiner's Use

Question No.	**Marks Awarded**
1	**/ 10**
2	**/ 9**
3	**/ 11**
4	**/ 9**
5	**/ 11**
Total	**/ 50**

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.81\text{ m s}^{-2} = \lvert\vec{g}\rvert$
Radius of the Earth,	$R_E = 6.360 \times 10^6\text{ m}$
Universal gas constant,	$R = 8.31\text{ J mol}^{-1}\text{ K}^{-1}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2\text{ N}^{-1}\text{ m}^{-2}$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ T m A}^{-1}$
Atomic mass unit,	$u = 1.66 \times 10^{-27}$ kg
Speed of light in vacuum,	$c = 3.00 \times 10^8\text{ m s}^{-1}$
Charge of electron,	$e = 1.60 \times 10^{-19}$ C
Planck's constant,	$h = 6.63 \times 10^{-34}$ J s
Mass of electron,	$m_e = 9.11 \times 10^{-31}\text{ kg} = 0.000549u$
Mass of proton,	$m_p = 1.67 \times 10^{-27}\text{ kg} = 1.007u$
Rest mass of alpha particle,	$m_\alpha = 4.003$u
Boltzmann constant,	$k = 1.38 \times 10^{-23}\text{ J K}^{-1}$
Avogadro's number,	$N_A = 6.02 \times 10^{23}\text{ mol}^{-1}$
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5$ Pa
Density of water,	$\rho_w = 1000\text{ kg m}^{-3}$
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3\text{ J kg}^{-1}\text{ K}^{-1}$
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5\text{ J kg}^{-1}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8}\text{ W m}^{-2}\text{ K}^{-4}$
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0+a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$
Area under the curve of $y = x^n$ for x between 0 and x_0	$\int_0^{x_0} x^n\, dx = \frac{1}{n+1} x_0^{n+1}$

1. (a) A cannon is placed at the edge of a vertical cliff. The muzzle of the cannon is 200 m above the level of sea water. It fires a shell with a muzzle velocity of 180 ms^{-1}. What should be the angle of elevation of the barrel of the cannon if the shell were to hit a stationary target at sea level and **a**t a perpendicular distance of 2.5 km away from the vertical wall of the cliff?

[3 marks]

1. (b) At the instant when the shell is fired, the target starts to move away from the cliff along a line perpendicular to the face of the cliff, with a constant speed of 10 ms^{-1}. Suppose the angle of elevation of the barrel of the gun remains unchanged, what alteration must be made to the muzzle speed of the shell in order for it to hit the target?

[7 marks]

2. (a) Two satellites X and Y are orbiting around Earth in circular orbits above the equator (or co-centre with the equator). The orbit of X is 500 km above the surface of Earth while that of Y is 1000 km above the surface Earth. At a particular instant, both satellites are vertically above a point on the surface of Earth. After t **minutes**, they are found to be vertically above another point on the surface of Earth. Determine the value of t, taking the radius of Earth to be 6360 km.

[3 marks]

2. (b) A sound source S and a detector D are placed at a distance of 120 m apart as shown in Fig. 1. The source S produces sound waves with wavelength 1.33 m. A reflector is placed so that its plane is parallel to the line joining S and D. When the reflector is situated at position 1 (see Fig.1), the direct wave from S is found to be in phase at the detector D with the wave from S that is reflected from the reflector. The incident and reflected waves make the same angle with the reflector. When the reflector is moved slowly away from position 1, the sound detected at D decreases in intensity and becomes zero for the first time when the detector reaches position 2 that is at a distance h from position 1. Determine the value of h.

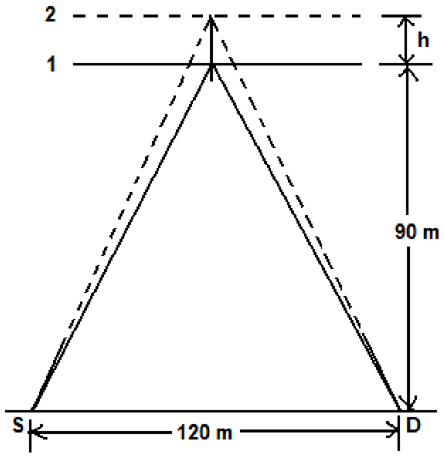
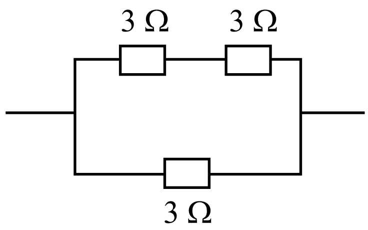


Fig. 1.

[6 marks]

3. (a) Three resistors, each of value 3 Ω, are arranged in two different arrangements as shown in Fig. 2. If the maximum allowable power for each individual resistor is 108 W, calculate the maximum power that can be dissipated by the circuits shown in Fig. 2(a) and 2(b).

2(a)



2(b)

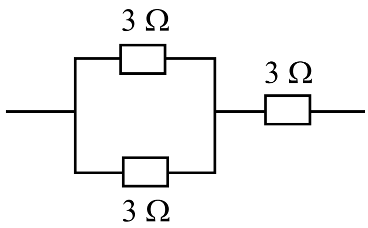


Fig. 2.

[4 marks]

3. (b) An insulating sphere with a diameter of 1.0 m carries a uniform charge density of 5 nC m $^{-3}$ throughout its entire volume. A small tunnel is drilled along a diameter of the sphere. A small particle with mass 1.0 μg and carrying charge -0.05 nC is placed at one end of the tunnel and released. Calculate:

(i) the time taken by the particle to move to the other end of the tunnel.

(ii) the speed of the particle when it reaches the centre of the sphere.

[7 marks]

4. (a) 1 mole of a <u>diatomic</u> ideal gas has a pressure of 10^5 Pa and occupies a volume of 25 litres. It is heated at <u>constant pressure</u> so that its volume is doubled. What is the change in entropy of the gas during the process?

[3 marks]

4. (b) A vessel contains 1 mole of a certain gas maintained at fixed temperature T. Each of the gas particle can exist in one of four energy states with energies 0.0043 eV, 0.0129 eV, 0.0215 eV and 0.0301 eV respectively. According to the theory in classical statistical physics, in a large ensemble of particles, the probability, $p(E)$, that a particle possesses energy E at temperature T is:

$$p(E) = \frac{e^{-\frac{E}{kT}}}{Z}$$

where k is the Boltzmann constant and Z is the normalization constant. It is found that 63% of the gas particles in the vessel exists in the lowest energy state; i.e. 0.0043 eV and 23% of them occupies the first excited state; i.e. 0.0129 eV. Estimate the number of gas particles existing in the 0.0215 eV state.

[6 marks]

5. (a) According to Niel Bohr's theory, the energy of a hydrogen atom when it is in the stationary state with principal quantum number n is given by $E_n = -\frac{me^4}{8\varepsilon_0{}^2h^2}\frac{1}{n^2}$, where m & e are the mass and charge of an electron, ε_0 is the permittivity of free space and h is the Planck's constant.

A hydrogen atom is initially at rest. An electron in the hydrogen atom makes a transition from the state with $n = 3$ to the state with $n = 1$. Calculate the recoil speed and recoil energy of the hydrogen during the process.
[Mass of hydrogen atom is 1.66×10^{-27} kg]

[6 marks]

5. (b) An observer A in the inertial frame S sees a red flash of light at the origin at time $t = 0$ and a blue flash of light at $x = x_B$ km and time $t = 9.50\mu s$. Another observer B is situated at the origin of another inertial frame S' which moves in the direction of increasing x with a speed of v relative to S. The axes of the two inertial frames are parallel at all times and their origins coincide at $t = t' = 0$. If $v = 0.6c$, what is the range of the values of x_B so that observer B sees the blue flash before seeing the red flash?

[5 marks]