34th Singapore Physics Olympiad Theory Paper

Organised by

Institute of Physics

In conjunction with
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National University of Singapore

Ministry of Education Singapore

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Instructions to Candidates

1. This is a **three and half** hour paper.

2. This paper consists of **TEN (10)** questions printed on TWENTYSEVEN (27) pages. The last page is a general information sheet.

3. Attempt all questions.

4. Write your answers in the space provided.

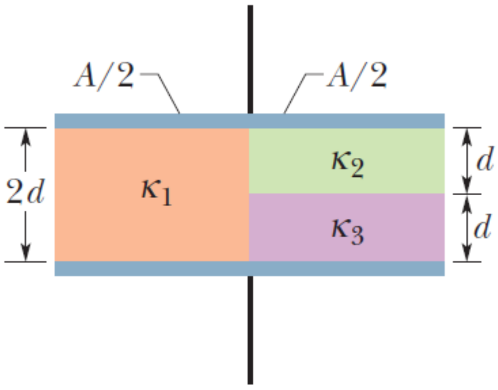
5. You may request working paper from the invigilators.

6. You may not refer to any books or documents relevant to the competition.

NAME: ____ (As in registration list)

SCHOOL: ____

1. A parallel-plate capacitor is shown in the diagram with plate area of A = 10.5 cm^2 and plate separation $2d$ = 7.12 mm. The left half of the gap is filled with material of dielectric constant κ_1 = 21.0; the top of the right half is filled with material of dielectric constant κ_2 = 42.0; the bottom of the right half is filled with material of dielectric constant κ_3 = 58.0. What is the capacitance?



[4 marks]

2. In the rectangle shown, the sides have lengths 5.0 cm and 15 cm, q_1 = -5.0 μC, and q_2 = +2.0 μC. With $V = 0$ at infinity, what is the electric potential at

(a) corner *A* and [1 mark]

(b) corner *B*? [1 mark]

(c) How much work is required to move a charge q_3 = +3.0 μC from *B* to *A* along a diagonal of the rectangle? [1 mark]

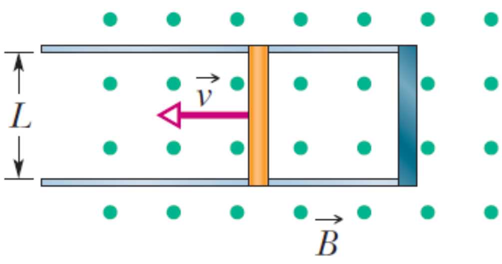
(d) Does this work increase or decrease the electric potential energy of the three-charge system? [1 mark]

Is more, less, or the same work required if q_3 is moved along a path that is

(e) inside the rectangle but not on a diagonal and [0.5 marks]

(f) outside the rectangle? [0.5 marks]

3. The conducting rod shown in the figure has length L and is being pulled along horizontal, frictionless conducting rails at a constant velocity $\vec{v}$. The rails are connected at one end with a metal strip. A uniform magnetic field $\vec{B}$, directed out of the page, fills the region in which the rod moves. Assume that L = 10 cm, v = 5.0 m/s, and B = 1.2 T. What are the

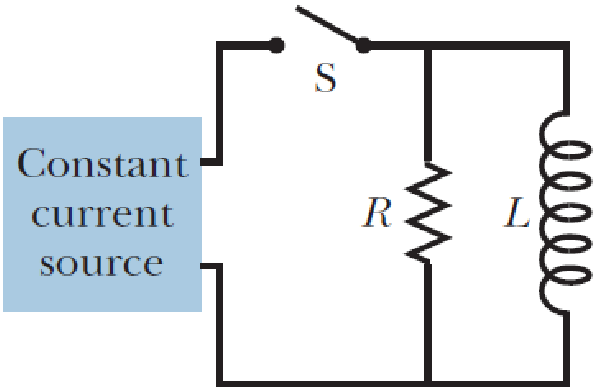


(a) magnitude and [1 mark]
(b) direction (up or down the page) of the emf induced in the rod? [0.5 marks]
(c) magnitude and [0.5 marks]
(d) direction of the current in the conducting loop? [0.5 marks]

Assume that the resistance of the rod is 0.40 Ω and that the resistance of the rails and metal strip is negligibly small.

(e) At what rate is thermal energy being generated in the rod? [1 mark]
(f) What external force on the rod is needed to maintain $\vec{v}$? [1.5 marks]
(g) At what rate does this force do work on the rod? [1 mark]

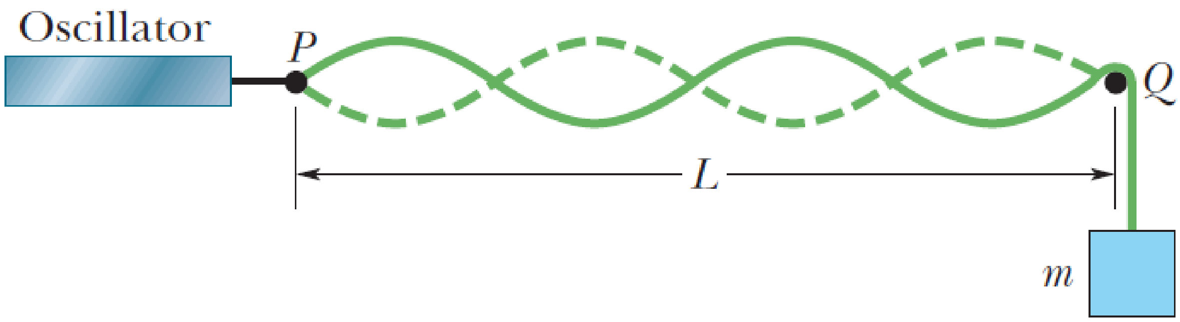
4. In the figure shown, after the switch S is closed at time $t = 0$, the emf of the source is automatically adjusted to maintain a constant current I through S.



(a) Find the current through the inductor as a function of time.
[4 marks]

(b) At what time is the current through the resistor equal to the current through the inductor?
[1 mark]

5. In the figure shown, a string, tied to a sinusoidal oscillator at P and running over a support at Q, is stretched by a block of mass m. The separation L between P and Q is 1.20 m, and the frequency f of the oscillator is fixed at 120 Hz. The amplitude of the motion at P is small enough for that point to be considered a node. A node also exists at Q. A standing wave appears when the mass of the hanging block is 286.1 g or 447.0 g, but not for any intermediate mass. What is the linear density of the string?

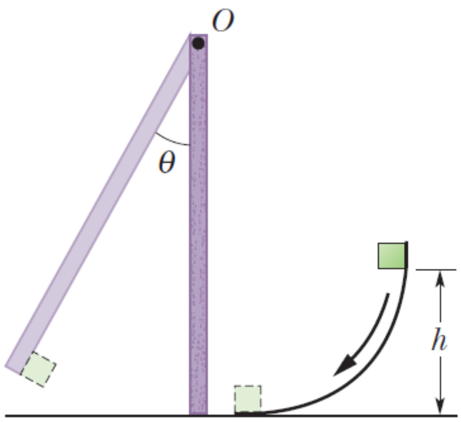


[6 marks]

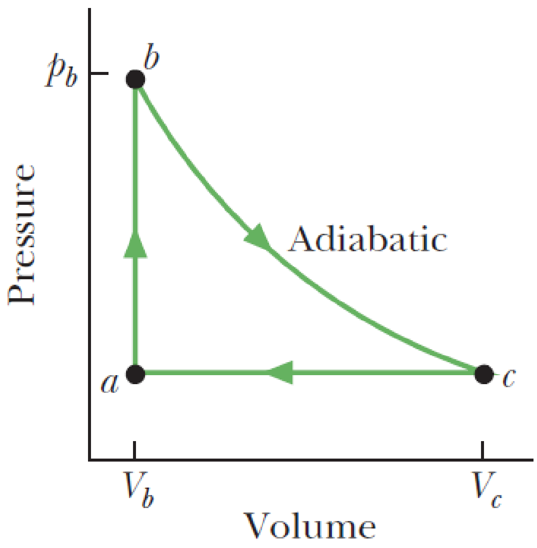
6. Calculate the minimum kinetic energy an electron must have in order to be constrained within the nucleus, with a typical nuclear radius of $r = 6 \times 10^{-15}$ m. Assume that the uncertainty of this electron's position is equal to the nuclear radius r. Treat the electron relativistically.

[6 marks]

7. A small 50 g block slides down a frictionless surface through height h = 20 cm and then sticks to a uniform rod of mass 100 g and length 40 cm. The rod pivots about point O through angle θ before momentarily stopping. Find θ.
[10 marks]



8. The figure shows a reversible cycle through which 1.00 mol of a monatomic ideal gas is taken. Volume $V_c = 8.00V_b$. Process bc is an adiabatic expansion, with $p_b = 10.0$ atm and $V_b = 1.00*10^{-3}$ m^3. For the cycle, find



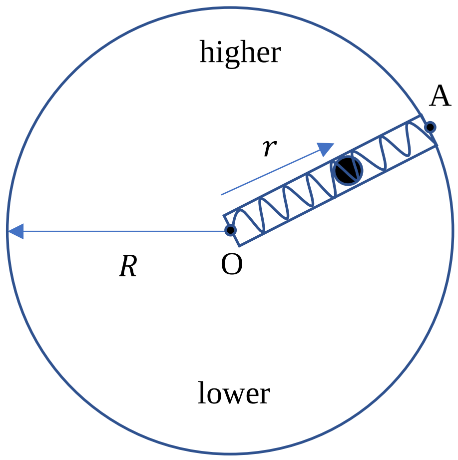
(a) the energy added to the gas as heat,
[3 marks]

(b) the energy leaving the gas as heat,
[3 marks]

(c) the net work done by the gas, and
[1 mark]

(d) the efficiency of the cycle.
[1 mark]

9. An ideal massless spring with spring constant $k = 10$ Nm^{-1} and unstretched length $L = 0.5$ m is attached to point O on axis of a disc. It has a point mass $m = 1\ kg$ attached at $L_1 = 0.2$ m of the unstretched spring. The spring is stretched and the other end is attached to point A on the edge of the disc at $R_0 = 1$ m. The point mass is constrained to **move radially only**.



The disc can rotate around the axis at a constant an angular frequency ω. Take $t = 0$ s when OA is horizontal and A is to the right and the disc is rotating anti-clockwise. Although some air resistance will help the system achieve a **steady state**, you may assume that air resistance is negligible in your working

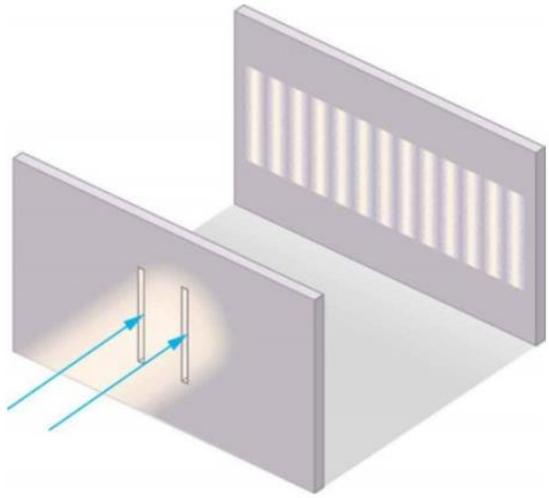
a) If the disc is **horizontal** and $\boldsymbol{\omega = 0}$**,** derive an expression for the **effective spring constant** k_t in terms of the given parameters.
[5 marks]

b) If the disc is **horizontal** and rotating and constant angular velocity ω, derive an expression for r_0, the **equilibrium** position of the mass.
[5 marks]

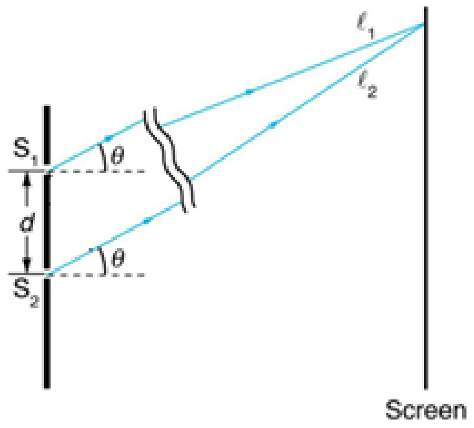
c) If the disc is at **an angle** $\boldsymbol{\varphi = 0.4}$ **rad** from the horizontal, what is the lowest angular frequency ω where the point mass can just about reach the edge of the disc at a **steady state**?
[5 marks]

10. Consider Young's double slit experiment as shown in the below figure (a). Monochromatic light is being used. Further assume that the screen distance ℓ is much larger than the slit separation d (Figure (b)).

(a)



(b)



(a) Under what condition does Young's experiment show interference pattern?
[0.5 marks]

(b) Determine the conditions for obtaining constructive and destructive interference.
[2 marks]

(c) Determine the distance Δy between adjacent fringes (maxima), assuming that θ (Figure (b)) is small.
[2 marks]

(d) Determine the intensity I of the diffraction pattern's maxima. You may use
$sin\alpha + sin\beta = 2cos\frac{1}{2}(\alpha - \beta)sin\frac{1}{2}(\alpha + \beta)$.
[4.5 marks]

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.80 \text{ m s}^{-2} = \lvert\vec{g}\rvert$
Radius of the Earth	$R_E = 6.371 \times 10^6 \text{m}$
Universal gas constant,	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$
Atomic mass unit,	$u = 1.66 \times 10^{-27} \text{ kg}$
Speed of light in vacuum,	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Charge of electron,	$e = 1.60 \times 10^{-19} \text{ C}$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J s}$
Mass of electron,	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.000549u$
Mass of proton,	$m_p = 1.67 \times 10^{-27} \text{ kg} = 1.007u$
Rest mass of alpha particle,	$m_\alpha = 4.003\text{u}$
Boltzmann constant,	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number,	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5 \text{ Pa}$
Density of water,	$\rho_w = 1000 \text{ kg m}^{-3}$
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N} a_k + (k-1)d = \frac{N}{2}\left[2a_0 + (N-1)d\right]$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N} r^k = \frac{1-r^N}{1-r} \qquad r < 1$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$