32nd Singapore Physics Olympiad Theory Paper 2

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Instructions to Candidates

1. This is a 2 hour paper.
2. This paper consists of five (5) questions printed on fifteen (15) pages. The last page is a general information sheet.
3. Attempt all questions.
4. Write your answers in the space provided in the question booklet.
5. You may request working paper from the invigilators.
6. You may not refer to any books or documents relevant to the competition.

NAME:____ INDEX NO: ___________

SCHOOL: _______________________________________

1. (a) A projectile is fired with initial speed of 50 ms^{-1} from the edge of a cliff which is at a vertical height of 100m from the sea-level. It hits a target which is at a horizontal distance of 300 m from the bottom of the cliff on the surface of the sea. What is the angle of inclination of the initial velocity of the projectile above the horizontal?

[4 marks]

1. (b) Suppose at the instant the projection, the target begins to move away from the cliff at a constant speed of 10 ms^{-1}. If the angle of projection remains the same as those computed in section (i), what must be the speed of projection so that the projectile hits the target? [8 marks]

2. (a)

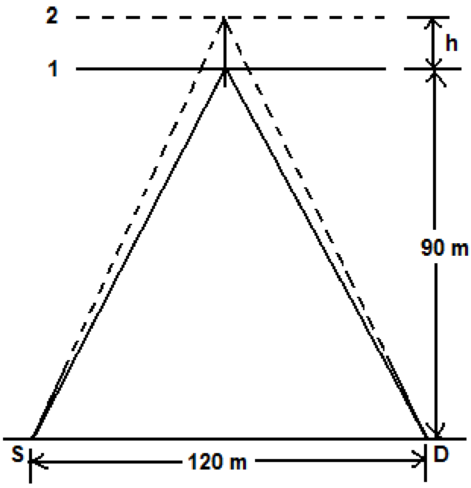

Fig. 1

A sound source S and a detector D are placed at a distance of 120 m apart as shown in fig. 2. The source S produces sound waves with wavelength 1.33 m. A reflector is placed so that its plane is parallel to the line joining S and D. When the reflector is situated at position 1, (see Fig.2) the direct wave from S is found to be in phase at the detector D with the waves from S that is reflected from the reflector. The incident and reflected waves make the same angle with the reflector. When the reflector is moved slowly away from position 1, the sound detected at D decreases in intensity, becomes zero, then increases and becomes maxima again for the first time when the detector reaches position 2 that is at a distance h from position 1. Determine the value of h. [5 marks]

2. (b) A sonometer wire has a diameter of 0.51 mm and is made of material with density 8.885 x 10^3 kg m^{-3}. It is stretched tightly over two bridges which are 60 cm apart. The tension in the wire is applied by hanging a uniform metal cylinder of diameter 5.0 cm and height 10.0 cm from the free end of the wire. The metal cylinder is slowly lowered into a liquid until half of its volume is immersed in the liquid. It is found that under this arrangement, when the sonometer wire is vibrating in its second harmonic mode, the frequency of vibration is 118.4 Hz. When the metal cylinder is completely immersed in the liquid, the frequency of vibration of the second harmonic is 114.7 Hz. Calculate the densities of the material of the cylinder and that of the liquid. [7 marks]

3. (a) The internal and external radii of a uniform hollow sphere are r and R respectively. Taking the gravitational potential to be zero at infinity, what is the ratio of the gravitational potential at a point on the outer surface to that on the inner surface?

[5 marks]

3. (b) (i) A well-lagged uniform cylindrical copper rod is 30.0 cm long and has a diameter of 2.00 cm. One end of it is in thermal contact with a hot reservoir maintained at a temperature of $200°C$ while the other end is in thermal contact with a cold reservoir at temperature $0°C$. Calculate the rate of change of entropy of the system comprising the hot and cold reservoirs and the copper rod.

[Thermal conductivity of copper = 400 W m $^{-1}$ K $^{-1}$] [3 marks]

3. b. (ii) An electron, travelling at a speed of 2.08 x 10^6 ms^{-1}, collides with a stationary hydrogen atom which is in a state in which its orbital angular momentum is $\hbar$. What are the possible wavelengths of the photons emitted by the hydrogen atom after collision?

[Energy level of the hydrogen atom $E_n = -\frac{13.6}{n^2}\ eV$, where n =1, 2, 3, 4,…]

[4 marks]

4. (a) A particle of mass m carries a negative charge of magnitude q It is placed at the centre of a uniformly charged ring, the axis of which is taken to be the x-axis. The radius of the ring is a and the linear charge density on the ring is λ. The particle is confined to move along the x-axis. It is displaced a **small** distance x along the axis (x<<a) and released. Show that the particle oscillates with simple harmonic motion along the x-axis and obtain an expression for the frequency of oscillation.

[The electric field at a point on the axis of a charged ring of radius r, distance x from the centre of the ring is $E = \frac{Q}{4\pi\varepsilon_0}\frac{x}{(a^2+x^2)^{3/2}}$ where Q is the total charge on the ring]

[6 marks]

4. (b) (i) According to Bohr's theory, when the hydrogen atom is in its ground state, the electron is revolving around the proton in circular orbit with a radius of 5.3×10^{-10} m. The speed of the electron in its orbit is 6.91×10^{5} ms^{-1}. Determine the magnitude of the magnetic field at the position of the proton due to the orbital motion of the electron.

(ii) A circular disc has a radius R and surface charge density σ. It is spinning at n revolutions per second. What is the magnetic field at the centre of the disc?

[6 marks]

5. (a) One event occurs at the origin of an inertial frame S at the time $t = 0$. Another event occurs at $x = 4c, y = z = 0, t = 5$ sec relative to S.

(i) Determine the velocity, relative to S, of the inertial frame S' in which the two events are recorded at the same point in space.

(ii) What is the time interval between the events in S'- frame? [6 marks]

(b) A cube with sides of length ℓ is moving with one of its sides parallel to the x - axis of an inertial frame S with velocity u. An observer is moving along the x - axis of the inertial frame S with velocity v. Both u & v are comparable to c, the speed of light. Derive an expression for the volume of the cube as measured by the observer in terms of ℓ, u, v & c. [6 marks]

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.81$ m s^{-2} $= \lvert\vec{g}\rvert$
Radius of the Earth,	$R_E = 6.371 \times 10^6$m
Universal gas constant,	$R = 8.31$ J mol^{-1} K^{-1}
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
Atomic mass unit,	$u = 1.66 \times 10^{-27}$ kg
Speed of light in vacuum,	$c = 3.00 \times 10^8$ m s^{-1}
Charge of electron,	$e = 1.60 \times 10^{-19}$ C
Planck's constant,	$h = 6.63 \times 10^{-34}$ J s
Mass of electron,	$m_e = 9.11 \times 10^{-31}$ kg $= 0.000549u$
Mass of proton,	$m_p = 1.67 \times 10^{-27}$ kg $= 1.007u$
Rest mass of alpha particle,	$m_\alpha = 4.003$u
Boltzmann constant,	$k = 1.38 \times 10^{-23}$ J K^{-1}
Avogadro's number,	$N_A = 6.02 \times 10^{23}$ mol^{-1}
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5$ Pa
Density of water,	$\rho_w = 1000$ kg m^{-3}
Specific heat (capacity) of water,	$c_w = 4.19 \times 103$ J kg^{-1} K^{-1}
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5$ J kg^{-1}
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10$-8 W m^{-2} K^{-4}
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0 + a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$
Area under the curve of $y = x^n$ for x between 0 and x_0	$\int_0^{x_0} x^n \, dx = \frac{1}{n+1} x_0^{n+1}$