31st Singapore Physics Olympiad
Theory Paper 2

Organised by

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Instructions to Candidates

1. This is a 2 hour paper.
2. This paper consists of four (4) questions printed on seventeen (17) pages.
3. Attempt all questions.
4. Write your answers in the space provided in the question booklet.
5. You may request working paper from the invigilators.
6. You may not refer to any books or documents relevant to the competition.

NAME:____

SCHOOL: _______________________________________

1. In the figure below, a singly ionized helium, with electron in its ground state i.e. n = 1, is at rest at position O. A neutron of kinetic energy of 68.2 eV is moving from the left towards O and collides inelastically with the singly ionized helium. The neutron is scattered at an angle of 90° with respect to its original direction of motion i.e. towards '$-y$' axis. The singly ionized helium after collision is excited and moves as is shown in the Figure 1. It may be noted that the mass of singly ionized helium is four times the mass of the neutron and the energy levels in the ionized helium atom are given by $E_n = -\dfrac{54.4}{n^2}$ eV .

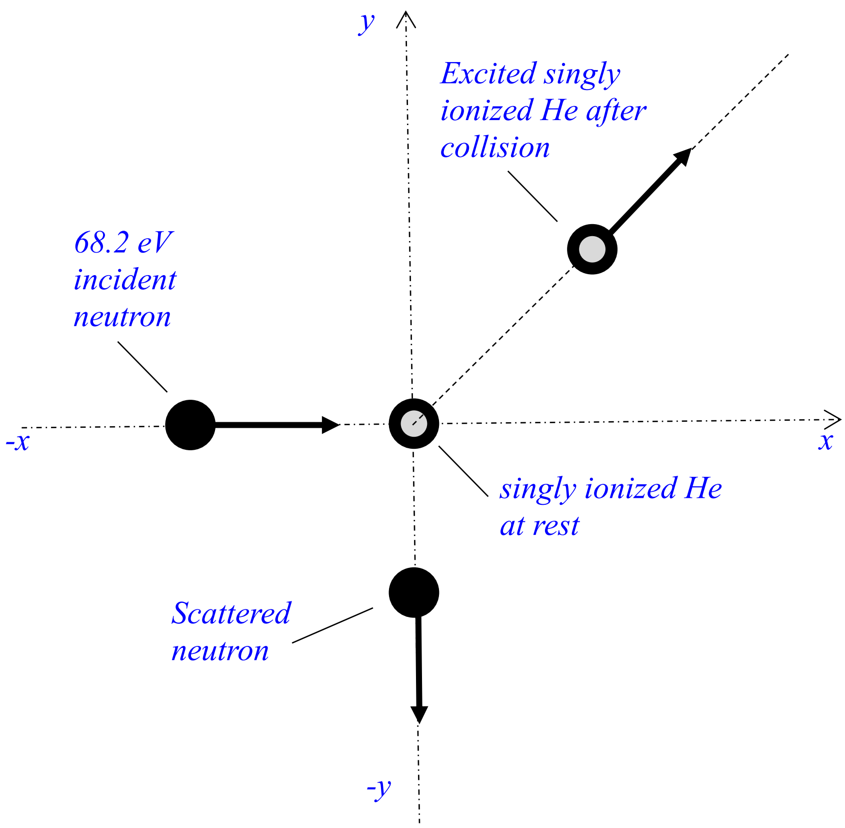

Fig. 1

(a) Find the allowed values of the kinetic energy of the scattered neutron and the kinetic energy of the single ionized helium after collision.

[8 marks]

(b) If the singly ionized helium get de-excited subsequently by emitting radiation, find the shortest and longest wavelengths of the emitted radiation. For this part assume that the momentum of the emitted photon is negligible.

[4 marks]

(c) When an atom emits a photon in a transition from the quantum state with energy E_i to the quantum state with energy E_f, the energy of the photon is not exactly equal to $E_i - E_f$. This is because the atom must recoil and thus some of the transition energy becomes the kinetic energy of recoil of the atom. Estimate the percentage of this transition energy, $E_i - E_f$, that becomes the recoil kinetic energy of the recoiling atom for the shortest wavelength case of the previous part.

[6 marks]

(d) Estimate also the change in speed of the helium ion due to this transition.

[2 marks]

2. A particle of mass 100 g is dropped from a great height and falls vertically downwards. The force due to air resistance is $-kv$ where v is the speed of the particle and k is a constant having the value 1.09×10^{-2} kg s^{-1}.

(a) What is the terminal velocity of the particle during the fall?

[1 mark]

(b) How **much time** does it take for the particle to achieve 99% of the terminal velocity?

[4 marks]

(c) How **far** has the particle fallen when its velocity achieves 99% of the terminal velocity?

[4 marks]

3. If a charged particle with charge q, mass m, having velocity $\vec{v} = (v_x, v_y, 0)$ travels in a magnetic field $\vec{B} = (0, 0, B_z)$ then a Lorentz force $\vec{F} = (F_x, F_y, 0)$ acts on the particle. As a result, the particle moves in a circular orbit. Assume a non-relativistic situation and that radiation is negligible.

(a) Derive an expression for T, the time taken for the charged particle to make one complete revolution in the orbit in terms of m, q, B_z.

[2 marks]

(b) Write down expressions for the acceleration of the particle in the x, y and z direction, i.e. $a_x(t)$, $a_y(t)$, $a_z(t)$ in terms of $v_x(t)$, $v_y(t)$ and T.

[3 marks]

(c) To compute the trajectory of the particle, one can estimate and compute the velocity of the particle after a short time step Δt. For method 1, to a first approximation, assume that the acceleration in the x and y directions are constant during this short period of time such that $v_x(t+\Delta t) = v_x(t) + a_x(t)\Delta t$. Write down equations which will allow $x(t+\Delta t)$ and $y(t+\Delta t)$ be calculated from $x(t)$ and $y(t)$ in terms of T, B_z, q, m & Δt.

[3 marks]

(d) Derive an equation for the change in kinetic energy ΔE_k after time Δt in terms of T, B_z, q, m & Δt.

[3 marks]

(e) Calculate the ratio $\frac{E_k'}{E_{k_0}}$ where E_k' is the kinetic energy at the end of an orbit and E_{k_0} is the initial kinetic energy if the time step Δt is $0.01T$.

[2 marks]

(f) Let us consider another way to compute the trajectory, method 2. We still assume that that the acceleration in the x and y directions are constant during this short period of time Δt. However for $t > 2\Delta t$, in our computation, we will approximate by making use of the average velocity from time $t - \Delta t$ to time $t + \Delta t$ when calculating the Lorentz force and work out the acceleration at time t as the change in average velocity $t - \Delta t$ to t and t to $t + \Delta t$. i.e.

$$v_x(t) = \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t}$$

$$a_x(t) = \frac{\frac{x(t+\Delta t) - x(t)}{\Delta t} - \frac{x(t) - x(t-\Delta t)}{\Delta t}}{\Delta t}$$

Obtain equations to calculate $x(t+\Delta t)$ and $y(t+\Delta t)$ from $x(t)$, $x(t-\Delta t)$ and $y(t)$, $y(t-\Delta t)$ in terms of B_z, q, m & Δt.

[6 marks]

(g) Derive an equation for the change in kinetic energy ΔE_k after time Δt in terms of B_z, q, m, Δt introduced by computing using method 2.

[4 marks]

(h) Sketch a diagram to illustrate the difference in the trajectory of the particle as computed using method 1 and method 2. Also illustrate the difference from the real trajectory.

[2 marks]

4. A rocket of proper length 600 m is moving directly away from the earth with uniform velocity. A radar pulse is sent out from the earth and is reflected from the reflectors at the back end and the front end of the rocket. The first reflected radar pulse is received back at the base 5.00 minutes after emission and the second reflected pulse is received 12.0 μs later.

(a) Calculate the distance of the rocket from the earth at the instant the outgoing radar pulse hits the back end reflector.

[2 marks]

(b) Calculate the velocity of the rocket relative to the earth.

[5 marks]

(c) Calculate the time interval between the reflections at the back end and front end of the rocket measured in the inertial frame of the rocket.

[3 marks]

(d) Explain why the time interval between the reflections in the two frames (i.e. the earth frame and the rocket frame) are not related by the time dilation formula.

[3 marks]

End of Paper