Thirtieth Singapore Physics Olympiad Theoretical Paper Part A

Saturday 21 October 2017

Organised by
Institute of Physics

In conjuction with
National University of Singapore
National Institute of Education, Nanyang Technological University,
Ministry of Education, Singapore

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Instructions to Candidates

1. This is a 1.5 hour paper.
2. This paper consists of four (4) questions printed on twelve (12) pages.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your answers in the space provided in the booklet.
5. If you need working paper, you may request from the invigilators.
6. No books or documents relevant to the test may be referred during the examination.

NAME: ____

SCHOOL: ____

1. [10 marks] A planet is in a circular orbit about a massive star of mass M. The star undergoes a spherically symmetric explosion where its outer envelope is ejected to a distance well beyond that of the planet's orbit. The remnant of the star has mass M' which is still much greater than the mass of the planet. Find the eccentricity of the new orbit of the planet. Assume that the mass loss is instantaneous and that the planet itself is unaffected by the explosion.

2. This question introduces the least time principle for light propagation, due to Pierre de Fermat, the 17th century French lawyer and mathematician who is also famous for his "last theorem".

The principle is that light "chooses" to take the path that allows it to go from one point to another in the shortest possible time, compared to any other path between those points.

Assume that the speed of light in a medium is inversely proportional to the refractive index of the medium.

To keep things simple, we will consider light confined in two dimensional space, so it only travels within a plane.

(a) [2 marks] Suppose that light must go from point A to point B in Figure 1. Using Fermat's principle, sketch the path on Figure 1. Label the path (a).

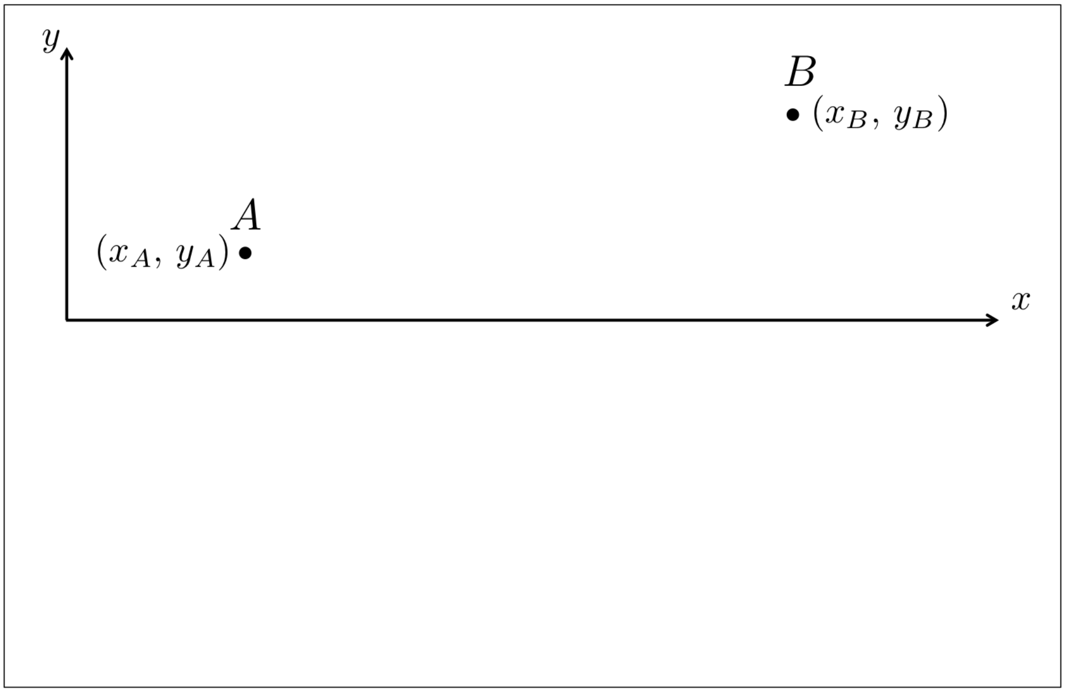


Figure 1: Path of light from A to B.

(b) Suppose that light must go from point A to point B, but must impinge upon the $x-$axis exactly once.

(i) [1 mark] For an arbitrary path, let the variable x be the x-coordinate of the intersection with the $x-$axis. Write down an expression for the length of an arbitrary path, in terms of x.

(ii) [3 marks] Hence use Fermat's principle to determine the actual path chosen and sketch on Figure 1. Label the path (b). Comment on the angles.

(c) [4 marks] Suppose that light must travel in two different media with refractive index $n_1 < n_2$, going from point A to point C in Figure 2. The y-axis sharply divides the two media. Use Fermat's principle to determine the path chosen. Comment on the angles.

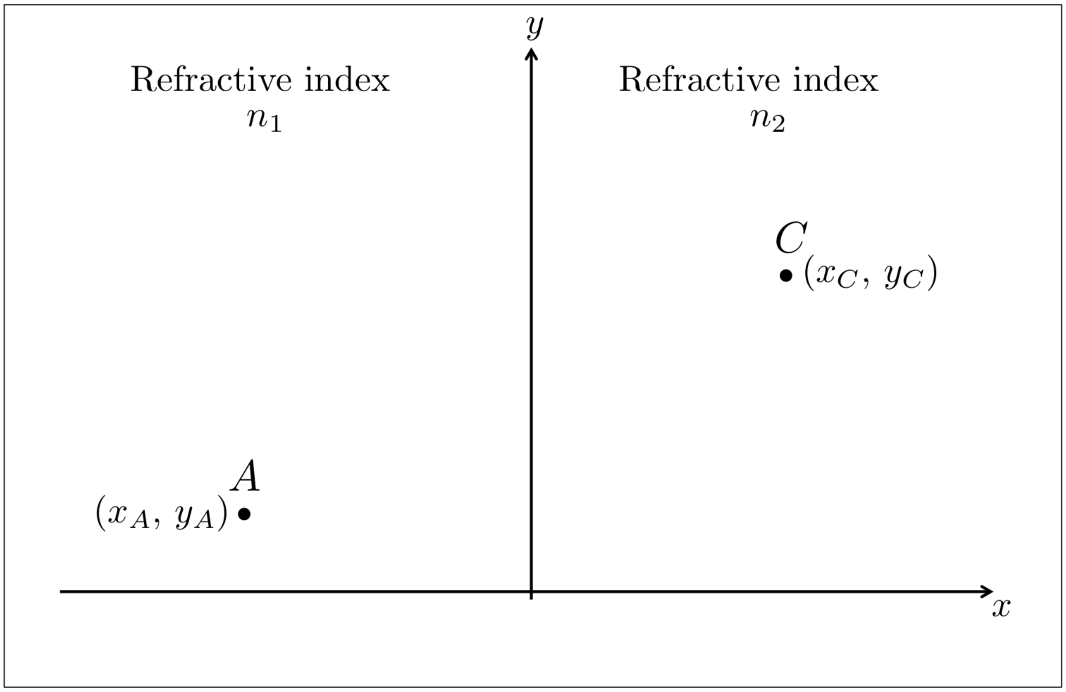
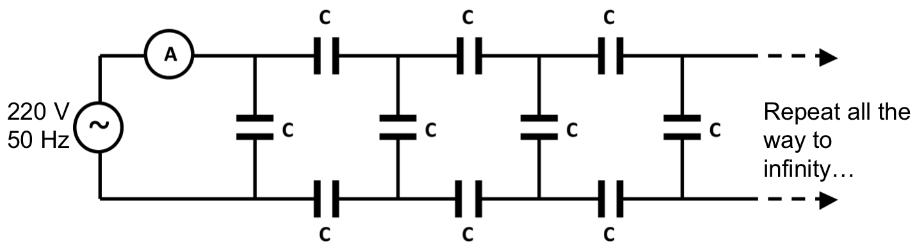


Figure 2: Path of light from A to C.

3. [10 marks] Consider the radioisotope ^{26}Cl. ^{26}Cl decays with 98.1% probability to ^{26}Ar and with 1.9% probability to ^{26}S. In a sample of old groundwater in a cave, the masses of ^{26}Cl and ^{26}S were measured to be 20 μg and 0.36 μg respectively. The age of this underground water was deduced to be 290 000 years old from the present. Find the half-life of ^{26}Cl.

4. [10 marks] An infinite capacitor network is connected to an AC voltage supply of 220 V, 50 Hz. The capacitance of each capacitor is $C = 1\mu\text{F}$.



Find the current through the ideal AC ammeter. Express your answer in terms of three significant figures.

General Data Sheet

Speed of light in vacuum	c	=	$299\,792\,458\ \mathrm{m \cdot s^{-1}}$
Vacuum permeability (magnetic constant)	μ_0	=	$4\pi \times 10^{-7}\ \mathrm{kg \cdot m \cdot A^{-2} \cdot s^{-2}}$
Vacuum permittivity (electrical constant)	ε_0	=	$8.854\,187\,817 \times 10^{-12}\ \mathrm{A^2 \cdot s^4 \cdot kg^{-1} \cdot m^{-3}}$
Elementary charge	e	=	$1.602\,176\,620\,8(98) \times 10^{-19}\ \mathrm{A \cdot s}$
Mass of the electron	m_e	=	$9.109\,383\,56(11) \times 10^{-31}\ \mathrm{kg}$
		=	$0.510\,998\,946\,1(31)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Mass of the proton	m_p	=	$1.672\,621\,898(21) \times 10^{-27}\ \mathrm{kg}$
		=	$938.272\,081\,3(58)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Mass of the neutron	m_n	=	$1.674\,927\,471(21) \times 10^{-27}\ \mathrm{kg}$
		=	$939.565\,413\,3(58)\ \frac{\mathrm{MeV}}{\mathrm{c^2}}$
Unified atomic mass unit	u	=	$1.660\,539\,040(20) \times 10^{-27}\ \mathrm{kg}$
Rydberg constant	R_∞	=	$10\,973\,731.568\,508(65)\ \mathrm{m^{-1}}$
Universal constant of gravitation	G	=	$6.674\,08(31) \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$
Acceleration due to gravity (in Zurich)	g	=	$9.81\ \mathrm{m \cdot s^{-2}}$
Planck's constant	h	=	$6.626\,070\,040\ (81) \times 10^{-34}\ \mathrm{kg \cdot m^2 \cdot s^{-1}}$
Avogadro number	N_A	=	$6.022\,140\,857\ (74) \times 10^{23}\ \mathrm{mol^{-1}}$
Molar gas constant	R	=	$8.314\,4598(48)\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot mol^{-1} \cdot K^{-1}}$
Molar mass constant	M_u	=	$1 \times 10^{-3}\ \mathrm{kg \cdot mol^{-1}}$
Boltzmann constant	k_B	=	$1.380\,648\,52(79) \times 10^{-23}\ \mathrm{kg \cdot m^2 \cdot s^{-2} \cdot K^{-1}}$
Stefan-Boltzmann constant	σ	=	$5.670\,367\ (13) \times 10^{-8}\ \mathrm{kg \cdot s^{-3} \cdot K^{-4}}$