Twenty-Sixth Singapore Physics Olympiad
Theoretical Paper

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Instructions to Candidates

1. This is a four-hour test.

2. This paper consists of **TEN** (10) questions printed on **SIX** (6) printed pages. Page **SIX** (6) is a Table of Fundamental Constants in Physics which may be useful for your calculations.

3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].

4. Write your name legibly on the top right hand corner of every answer sheet you submit.

5. Begin each answer on a fresh sheet of paper.

6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.

7. No books or documents relevant to the test may be brought into the examination hall.

1. The first explosion of the atomic bomb was conducted near Alamogoro, New Mexico on July 16, 1945. Figure 1 shows a picture of the approximately spherical shockwave 25 msec with the tick marks indicating 100 m Using dimensional analysis, find an
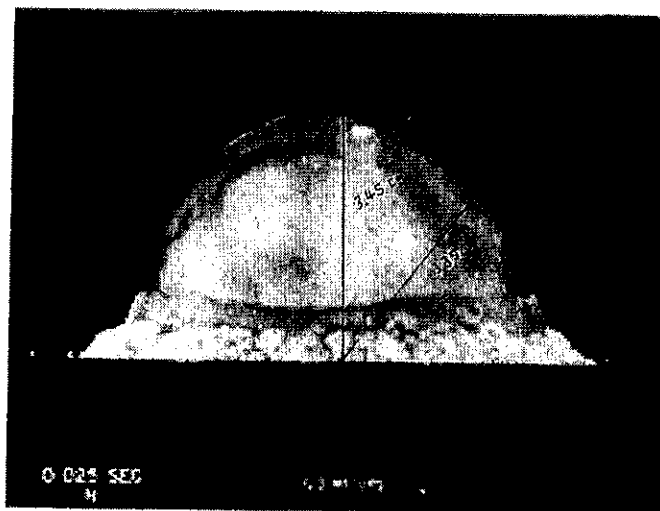

Figure 1: Camera shot of the bomb after 25 msec.

expression for the radius of the shockwave, R, as a function of the density ρ of the medium (i.e. the air) through which the shock wave travels, the energy of the explosion, E and the time, t, that elapsed since the explosion.

Assuming that the proportionality constant is unity (i.e. 1), as predicted by Sir Geoffrey Taylor, and assuming that the density of air, ρ, is 1.2 kg m^{-3}, and that the explosion of 1000 kg of TNT releases an energy of 4.184×10^9 J, estimate, from the picture, the approximate energy of this explosion expressed as tons (1000 kg) of TNT equivalent. [8]

2. During a discussion in a physics meeting, Einstein used this problem to illustrate the notion of adiabatic invariant. A pendulum, made up of a ball of mass M suspended from a pivot by a light string of length L, is swinging freely in a vertical plane. By what factor does the amplitude of the oscillations change if the string is shortened very slowly by a factor of 2? [10]

3. A rocket is projected straight up and explodes into three equally massive fragments just as it reaches the top of its flight. One of the fragments is observed to come straight down to the ground in time t_1 while the other two land at time t_2 after the burst. Find the height $h(t_1, t_2)$ at which the fragmentation occurred. [10]

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4. An early but unfortunately incorrect model of the hydrogen atom suggested by J.J. Thomson postulated that the hydrogen atom is a positive cloud of charge $+e$ uniformly distributed throughout a volume of a sphere of radius R with the electron represented by a equal magnitude but negative point charge $-e$ at the center of the sphere.

   (i) Show that the electron is in equilibrium at the center and that if displaced a small distance $r < R$ would experience a restoring force of the form $F = -Kr$ where K is a constant. Determine K. [3]

   (ii) Find an expression for the frequency of the simple harmonic oscillations that the electron of mass m_e would undergo if displaced a small distance from the center and released. [3]

   (iii) Calculate a numerical value for R that would result in a frequency of 2.47×10^{15} Hz, the frequency of light radiated in the most intense line of the hydrogen spectrum. [3]

5. A "perfect" rocket engine combines matter and antimatter in a controlled way to yield photons (high energy gamma rays) all of which are directed out of a spaceship. Suppose we start with a spaceship of initial mass M_0 at rest, and that at burnout, the remaining spaceship moves at speed v and has mass m equal to the fraction f of the original mass. You may take the speed of light $c = 1$ and $\gamma = 1/\sqrt{1 - v^2}$.

   (i) What is the total energy of this system initially? Let E_{rad} stand for the total energy of radiation after the burnout. Find an expression for the total energy of the system after the burnout and set up the conservation of energy equation. [2]

   (ii) Write down the equation for the conservation of momentum. [2]

   (iii) Show that $\gamma f + \gamma v f = 1$. [2]

   (iv) Show that $\gamma v = \sqrt{\gamma^2 - 1}$ so that $f^2 - 2\gamma f + 1 = 0$. [3]

   (v) What is the value of f if $\gamma = 10$? Is it possible to construct a spaceship whose shell and payload with this f? [3]

   (vi) Using f obtained above for $\gamma = 10$, show that the total energy of the emitted radiation is less than the mass of fuel consumed. Explain why. [3]

6. On a smooth and insulating large ring of radius R, there is a small ring of mass m and carrying charge q. The large ring is placed horizontally and in a uniform magnetic field of strength B_0 and perpendicular to the ring plane. Starting at $t = 0$, the magnetic field is changed to $B(t) = B_0 + \alpha t$. Find the force of the small ring acting on the big ring afterwards and describe the motion of the small ring. [10]

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7. Three polarizing disks whose planes are parallel are centered on a common axis. The direction of the transmission axis in each case is shown in Figure 2 relative to the common vertical direction. A plane-polarized beam of light with E0 parallel to the vertical reference direction is incident from the left on the first disk with intensity $I_0 = 10.0$ in some arbitrary units. Calculate the transmitted intensity If when

(i) $\theta_1 = 20.0°, \theta_2 = 40.0°$, and $\theta_3 = 60.0°$; [3]

(ii) $\theta_1 = 0°, \theta_2 = 30.0°$, and $\theta_3 = 60.0°$; [3]

(iii) If $\theta_1 = 0°$ and $\theta_3 = 60.0°$, find the angle θ_2 so that maximum intensity passes through the three polarizers. [4]
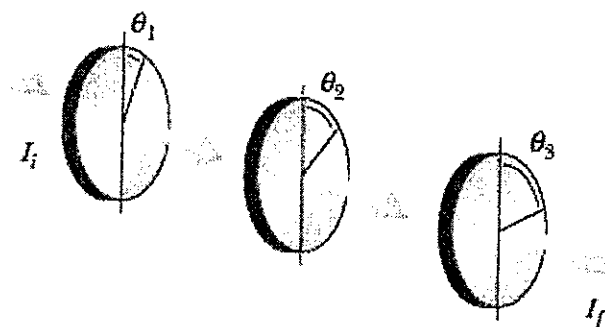

Figure 2: Three polarizing disks whose planes are parallel are centered on a common axis

8. Two parallel rails with negligible resistance are 10.0 cm apart and are connected by a 5.00 Ω resistor. The circuit also contains two metal rods having resistances of 10.0 Ω and 15.0 Ω sliding along the rails (see figure 3). The rods are pulled away from the resistor at constant speeds of 4.00 ms^{-1} and 2.00 ms^{-1}, respectively. A uniform magnetic field of magnitude 0.0100 T is applied perpendicular to the plane of the rails. Determine the current in the 5.00 Ω resistor. [8]
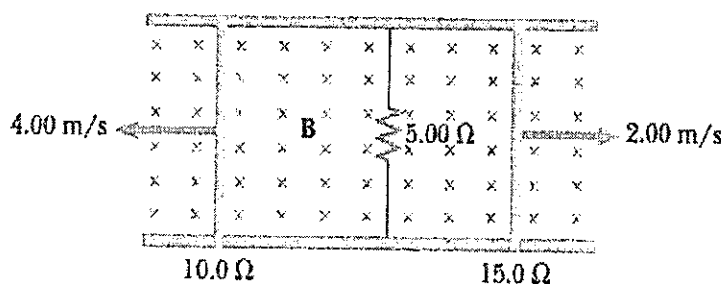

Figure 3: Two parallel rails with negligible resistance and connected by a resistor

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9. In Figure 4, the switch is closed for $t < 0$, and steady-state conditions are established. The switch is opened at $t = 0$.

   (i) Find the initial voltage $\mathcal{E}_t$ across L just after $t = 0$. Which end of the coil is at the higher potential: a or b? [3]

   (ii) Sketch graphs of the currents in R_1 and in R_2 as a function of time, treating the steady-state directions as positive. Show the values before and after $t = 0$.

   would I across [5]

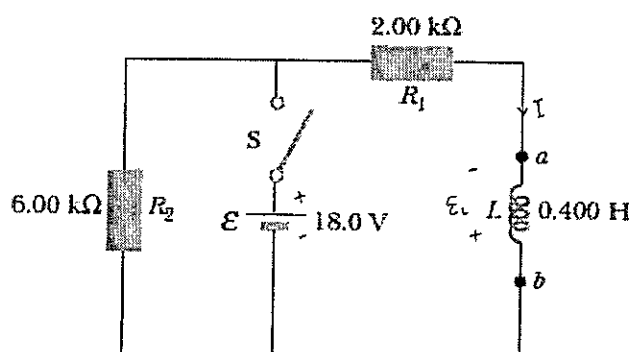   (iii) How long after $t = 0$ R_2 have the value 2.00 mA? [2]

Figure 4: Two parallel rails with negligible resistance and connected by a resistor

10. A transparent cylinder of radius $R = 2.00$ m has a mirrored surface on its right half, as shown in Figure 5. A light ray traveling in air is incident on the left side of the cylinder. The incident light ray and exiting light ray are parallel and $d = 2.00$ m. Determine the index of refraction of the material. [10]
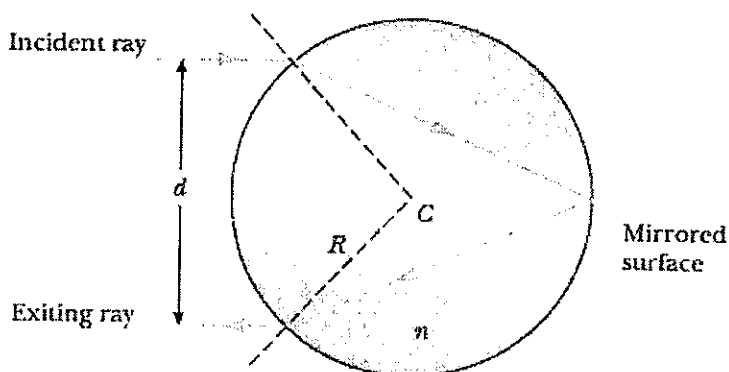

Figure 5: Transparent cylinder with mirrored surface

END OF PAPER

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational value
Avogadro's number	N	6.023×10^{23} mole^{-1}
Boltzmann constant	k	1.38×10^{-23} JK^{-1}
Elementary charge	e	1.6×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Neutron rest mass	m_n	1.68×10^{-27} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} Js
Permittivity constant	ϵ_0	8.85×10^{-12} Fm^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan's constant	σ	5.68×10^{-8} Wm^{-2} K^4
Speed of light in vacuum	c	2.997×10^8 ms^{-1}
Unified atomic mass unit	u	1.66×10^{-27} kg
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2}kg^{-2}
Mass of Earth	M_E	6×10^{24} kg
Radius of Earth	R_E	6.4×10^6 m
Mean Earth-Sun distance	R_{SE}	1.495×10^{11} m
Acceleration due to free fall	g	9.8 ms^{-1}