Twentieth-Second Singapore Physics Olympiad
Theoretical Paper

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Instructions to Candidates

1. This is a four-hour test.
2. This paper consists of **TEN** (10) questions printed on **SIX** (6) printed pages. Page **FIVE** (5) is a Table of Fundamental Constants in Physics which may be useful for your calculations. The last page is deliberately left blank.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your name legibly on the top right hand corner of every answer sheet you submit.
5. Begin each answer on a fresh sheet of paper.
6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.
7. No books or documents relevant to the test may be brought into the examination hall.

1. A small sphere is released from rest and, after falling through a vertical height of distance 1 m, bounces on a smooth plane inclined at an angle of $5°$ to the horizontal. If the sphere does not lose energy during the impact, determine the direction of motion immediately after impact with the plane and find the distance, measured down the plane, between this impact and the next one. [10]

2. A solid sphere of radius R rolls without slipping in a cylindrical trough of radius $5R$ as shown in Figure 1. Show that, for small displacements from equilibrium perpendicular to the length of the trough, the sphere executes simple harmonic motion. Determine the period of the simple harmonic motion. [8]

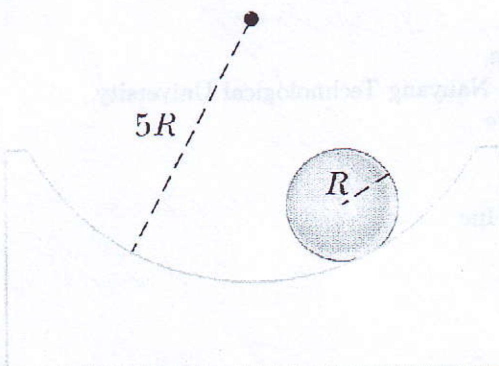

Figure 1: Solid sphere in a cylindrical trough.

3. A illuminating point object, a converging lens and a convex (diverging) mirror are arranged coaxially so that light from the object passes through the lens reflects at the mirror and returns through the lens. The mirror and the lens are both of focal length 20 cm and the lens is 40 cm from the object. Determine the position and nature of the final image formed when the mirror is

   (i) in contact (touching) the lens,

   (ii) 40 cm from the lens

   Illustrate your answers with ray diagrams. [8]

4. A microwave detector is located on the shore of a large lake 0.50 m above the water level. When a radio star emitting microwaves of frequency 1.5×10^9 Hz first appears on the horizon across the lake, the signal from the detector is very small. As the star rises, the signal increases, passes through a maximum and then falls again. If the signal is assumed to be a minimum when the star first appears, at what angle above the horizon is the star when the next minimum is recorded. [5]

5. The workfunction of potassium is 2.25 eV. A beam of light of wavelength 400 nm has an intensity of 3×10^{-9} Wm^{-2}. Find

   (i) the maximum kinetic energy of the photoelectrons, and [3]

   (ii) the number of electrons emitted per meter squared per second from the surface, assuming that only 3% of the incident photons are effective in ejecting electrons. [4]

6. An object is fired vertically upward from the surface of the Earth (of radius R_E) with an initial speed v_i that is comparable to but less than the escape speed v_{esc}.

   (i) Show that the object attains a maximum height h given by

$$h = \frac{R_E v_i^2}{v_{esc}^2 - v_i^2} \tag{1}$$

[3]

   (ii) A space vehicle is launched vertically upward from the Earth's surface with an initial speed of 8.76 km s^{-1}, which is less than the escape speed of 11.2 km s^{-1} Find the maximum height that it attains. [3]

   (iii) A meteorite falls toward the Earth. It is essentially at rest with respect to the Earth when it is at a height of 2.51×10^7 m. Determine the speed at which the meteorite strikes the Earth. [4]

   (iv) Assume that a baseball is tossed up with an initial speed that is very small compared to the escape speed. Show that maximum height attained by the baseball is given by

$$h = \frac{v_i^2}{2g} \tag{2}$$

   where g is the acceleration due to free fall on the surface of the Earth. Show also that the equation (1) from part (a) is consistent with this equation (2). [4]

7. Two ideal cells and three resistors with resistances 2.00, 5.00 and 7.00 Ω are connected as shown in Figure 2. If the ammeter reads 2 A, determine the currents I_1, I_2 and $\mathcal{E}$. [9]

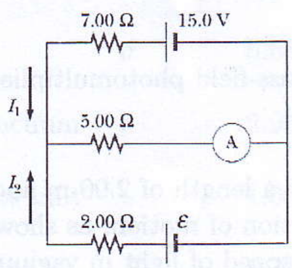

Figure 2: DC Circuit

8. A certain amount of steam initially at 130°C is used to warm 200 g of water in a 100-g glass container from 20.0°C to θ . Calculate the mass of steam needed if the temperature

   (i) $\theta = 100°$ C;

   (ii) $\theta = 50°$ C.

[9]

You may use the following data:

Specific heat capacity of glass	837 Jkg^{-1} K^{-1},
Specific heat capacity of steam	2.01 $\times 10^3$ Jkg$^{-1}K^{-1}$,
Specific heat capacity of water	4.19 $\times 10^3$ Jkg$^{-1}K^{-1}$,
Specific latent heat of vaporization of water	2.26 $\times 10^6$ Jkg^{-1}.

9. Figure 3 shows the principle of operation of a crossed-field photomultiplier. A sealed and evacuated enclosure contains two parallel plates called dynodes. These plates provide the electric field $\vec{E}$. An external permanent magnet superimposes a magnetic field $\vec{B}$. A photon ejects a low energy photoelectron. The electron accelerates upwards under the electric field but it is deflected to the negative dynode by the magnetic field. On impact with the dynode it ejects a few secondary electrons and the process repeats itself until eventually the electrons impinge on a collector C. Assuming that there is a steady voltage V between the plates and that the first photoelectron is emitted with zero initial speed, i.e. $\dfrac{dy}{dt} = \dfrac{dx}{dt} = 0$ at $x = y = 0$,

(i) write down differential equaion to describe the horizontal and vertical velocity of the electrons as a function of x and y, [8]

(ii) solve the differential equations for $x(t)$ and $y(t)$ (this trajectory is a cycloid), [4]

(iii) find the maximum value of y, and [4]

(iv) determine the value of a. [4]

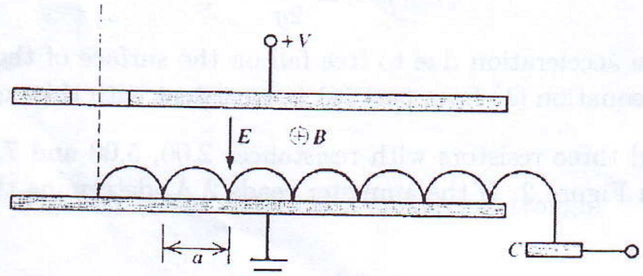

Figure 3: Cross-field photomultiplier.

10. A moving rod is observed to have a length of 2.00 m and to be oriented at an angle of 30.0° with respect to the direction of motion, as shown in Figure 4. The rod has a speed of 0.995c, where c is the speed of light in vacuum.

(i) What is the proper length of the rod? [5]

(ii) What is the orientation angle in the proper frame? [5]

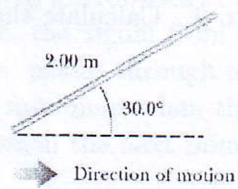

Figure 4: Moving rod

END OF PAPER

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational value
Avogadro's number	N	6.023×10^{23} mole^{-1}
Boltzmann constant	k	1.38×10^{23} JK^{-1}
Elementary charge	e	1.6×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Neutron rest mass	m_n	1.68×10^{-27} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} Js
Permittivity constant	ϵ_0	8.85×10^{-12} Fm^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan's constant	σ	5.68×10^{-8} Wm^{-2} K^4
Speed of light in vacuum	c	2.997×10^8 ms^{-1}
Unified atomic mass unit	u	1.66×10^{-27} kg
Universal gas constant	R	8.31 J mol^{-1} K^{-1}
Gravitational constant	G	6.67×10^{-11} N m^{2}kg^{-2}
Mass of Earth	M_E	6×10^{24} kg
Radius of Earth	R_E	6.4×10^6 m
Mean Earth-Sun distance	R_{SE}	1.495×10^{11} m