Nineteenth Singapore Physics Olympiad
Theoretical Paper

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Organized by

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Instructions to Candidates

1. This is a four-hour test.
2. This paper consists of **TEN** (10) questions printed on **SIX** (6) printed pages. Page **FIVE** (5) is a Table of Fundamental Constants in Physics which may be useful for your calculations. The last page is deliberately left blank.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your name legibly on the top right hand corner of every answer sheet you submit.
5. Begin each answer on a fresh sheet of paper.
6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.
7. No books or documents relevant to the test may be brought into the examination hall.

1. The center of gravity of an empty cylindrical beaker of mass m, radius r and negligible wall thickness is located at a height h above its base. Find the depth x that it should be filled with a liquid of density ρ so that it is made as stable as possible in terms of the given parameters. Find also the center of gravity of the partially filled beaker, y, above its base. What is the relation between x and y? [6 marks]

2. A light rope carries a mass of 100 kg on one end and is wound around a horizontal cylindrical bar with the coefficient of static friction μ between the rope and the bar being 0.05.

   (a) By considering infinitesimal portion of the rope, show that the force on the free end needed to support the mass depends exponentially on the number of turns of the rope in contact with the bar. [8 marks]

   (b) Find the minimum number of turns required if the only available counterweight has mass 1 kg. [5 marks]

3. A rain drop is falling through a cloud of small water droplets. Some of the water droplets adhere to the rain drop, adhering to the falling rain drop and increasing its mass. If the mass of the falling drop is proportional to the distance x that it travels in the cloud, show that

$$x\frac{d^2x}{dt^2} + \left(\frac{dx}{dt}\right)^2 = xg$$

   where g is the acceleration due to free fall, and t is the time of motion. [5 marks] Given that the proportionality constant for the mass is $k = 2$ g m^{-1}, find the mass of the rain drop at $t = 0.5$ s. [9 marks]

4. A man stands on a long plane concrete runway above which a uniform vertical temperature gradient results in a uniform gradient in the refractive index of the air, $n(x, y) = n_0(1 + \alpha y)$, where $|\alpha| = 1.5 \times 10^{-6}$ m^{-1}. As a result he cannot see the runway ($y = 0$) beyond a certain distance d. If his eyes are 1.7 m above the runway, find the value of d. Does the temperature rise or fall with increasing height? [8 marks]

5. Two parallel conducting plates each of area A and with separation d are connected to a source of constant voltage V. The plates are allowed to approach each other gradually until their separation is $\frac{1}{3}d$. The source is then disconnected and the separation of the plates is slowly restored to the value d. What difference, if any, is there between the initial and final electrostatic energies stored in the capacitors? [6 marks]

6. An aeroplane flies at a constant speed V relative to the air and completes a level circular course in time T on a windless day. If there is a steady wind of speed kV blowing in a fixed horizontal direction, determine the increase (or decrease) in the time needed for the course, assuming at $k << 1$, in terms of k and T. [8 marks]

7. A bicycle dynamo consists of a small permanent magnet which is fixed at its center to the axle of one of the bicycle wheels, which itself has radius r. Flux from the magnet links (via an iron yoke) a coil of self-inductance L, the coil being connected to a lamp of resistance R. The flux Φ linking the coil can be approximated as a sinusoidally varying quantity of the form $\Phi_0 \cos(\omega t)$.

   (i) Establish the equations satisfied by I_0 and ϵ, if the current in the circuit is $I_0 \cos(\omega t + \epsilon)$, and show that $\tan \epsilon = \dfrac{Rr}{vL}$, where v is the road-speed of the bicycle. [8 marks]

   (ii) Find how the power P delivered to the lamp varies with v, and show that it cannot exceed $\dfrac{R\Phi_0^2}{2L^2}$. [5 marks]

8. In the liquid drop model proposed by Weizsacker, the mass of the nucleus of mass number A and atomic number Z is given by

$$M = Zm_p + (A - Z)m_n - a_1 A + a_2 A^{2/3} + a_3 \frac{(A - 2Z)^2}{A} + a_4 \frac{Z^2}{A^{1/3}}$$

   where m_p and m_n are the masses of the proton and neutron respectively and a_i are positive constants.

   (i) Show that the nucleide with the greatest mass has atomic number Z given by

$$\frac{Z}{A} = \frac{m_n - m_p + 4a_3}{8a_3 + 2a_4 A^{2/3}}.$$

   [4 marks]

   (ii) Assume that the result in (i) can be approximated as $Z = \lambda A (\lambda \approx 0.46)$ and that the mass of neutron is the same as the mass of proton.

      (a) By considering the mass per nucleon, show that the nucleus with maximum value of mass per nucleon has atomic mass given by

$$A_s \approx \frac{a_2}{2\lambda^2 a_4}$$

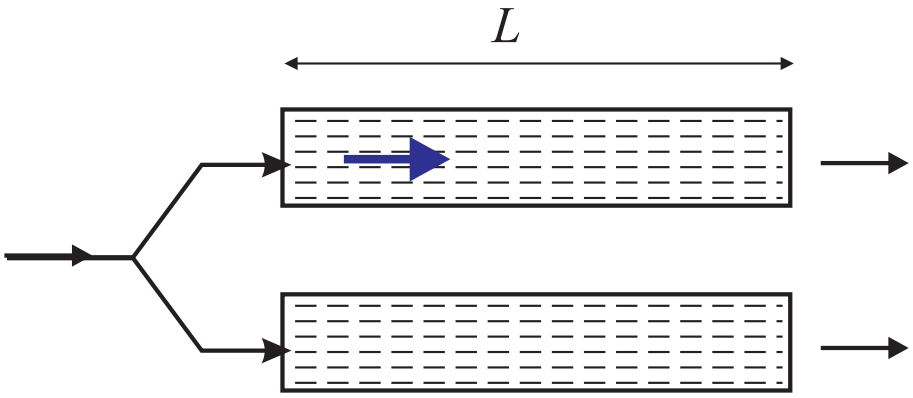
      . [4 marks]

      (b) Show that the model predicts that, since iron is the element with the largest binding energy per particle, spontaneous fission into two equal fragments should occur for elements with mass numbers higher than that of bromine. [8 marks]

   You may wish to use the following information:

Atomic number of iron	26
Atomic mass of iron	56
Atomic number of bromine	35
Atomic mass of bromine	79

9. A beam of monochromatic light, whose wavelength and speed in free space is λ and c respectively, is split into two separate beams and each of them is the passed through identical troughs of water of length L. If the water in one trough is stationary and the water in the other trough is moving with speed $\nu(<< c)$ in the direction of the light, determine the phase difference in terms of L, λ, ν, c and n, where n is the refractive index of the stationary water. [8 marks]



10. A photon of frequency ν is scattered, after colliding with an electron initially at rest through an angle of 90°. Show that its frequency ν' after being scattered is given by

$$\nu' = \frac{m_e c^2}{h\nu + m_e c^2}\nu.$$

Determine the kinetic energy of the recoiling electron after the scattering if the X-ray photon has initial energy 50.0 keV. [8 marks]

END OF PAPER

SOME FUNDAMENTAL CONSTANTS OF PHYSICS

Constant	Symbol	Computational value
Avogadro's number	N	6.023×10^{23} mole^{-1}
Boltzmann constant	k	1.38×10^{23} JK^{-1}
Elementary charge	e	1.6×10^{-19} C
Electron rest mass	m_e	9.11×10^{-31} kg
Neutron rest mass	m_n	1.68×10^{-27} kg
Proton rest mass	m_p	1.67×10^{-27} kg
Planck's constant	h	6.63×10^{-34} Js
Permittivity constant	ϵ_0	8.85×10^{-12} Fm^{-1}
Permeability constant	μ_0	$4\pi \times 10^{-7}$ Hm^{-1}
Stefan's constant	σ	5.68×10^{-8} Wm^{-2} K^4
Speed of light in vacuum	c	2.997×10^8 ms^{-1}
Unified atomic mass unit	u	1.66×10^{-27} kg
Universal gas constant	R	8.31 J mol^{-1} K^{-1}

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