Seventeenth Singapore Physics Olympiad
Theoretical Paper

Thursday, 28 October 2004

Organized by

Institute of Physics, Singapore

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Instructions to Candidates

1. This is a four-hour test.
2. This paper consists of **THREE** (3) questions printed on **FOUR** (4) pages.
3. Attempt all questions. Marks allocated for each part of a question are indicated in the brackets [ ].
4. Write your name legibly on the top right hand corner of every answer sheet you submit.
5. Begin each answer on a fresh sheet of paper.
6. Submit all your working sheets. No paper, whether used or unused, may be taken out of this examination hall.
7. No books or documents relevant to the test may be brought into the examination hall.

1. (a) A cannon fires a shot at an angle of ϕ up a plane inclined to the horizontal at angle θ as shown in Fig 1. In terms of θ, at what angle, ϕ, should the shot be fired in order to maximize the distance s up the inclined plane? Explain clearly how you obtain your answer. [8 marks]

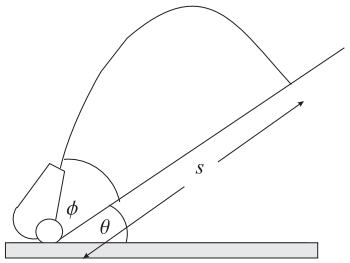

Figure 1: Cannon firing shot up an inclined slope.

(b) In its rest frame, a source emits light in a conical beam of width $\pm 60°$. For a the frame moving towards the source at high speed v, comparable to the speed of light c, the beam width is $\pm 45°$. Determine the value of v. [8 marks]

(c) A telephone wire of diameter 1 mm is suspended parallel to the ground at a height of 10 m. Determine the capacitance to the ground of this wire per unit length. (You may assume the ground to be a conducting plane). [7 marks]

(d) (i) Explain why the decay of radioactive material obeys an exponential law.

(ii) A age of a piece of wood is determined by measuring its radioactivity of carbon-14. The proportion of carbon-14 to carbon-12 in living wood is about 1.25×10^{-12}. When the piece of wood dies, the carbon-14 decays with a half life of 5600 years. If the number of disintegrations measured from 10.0 g of carbon prepared from a piece of wood is 48 per minute, estimate the age of the wood (in years). [7 marks]

(e) The lowest temperature in outer space (cosmic microwave background radiation) is about 2.7 K. Determine the root mean square speed of hydrogen molecules at this temperature. Molecular motions are not maintained by external forces, and yet continue indefinitely with no sign of diminishing speed. Why does friction or collision not bring these tiny particles to rest? [5 marks]

1 year	3.16×10^7 s
Avogadro's number	$\mathrm{N} = 6.023 \times 10^{23}$ mole^{-1}
Proton mass	$m_p = 1.67 \times 10^{-27}$ kg
Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J K^{-1}
Permittivity constant	$\epsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}
Speed of light	$c = 2.997 \times 10^8$ ms^{-1}

2. (a) A peacock feather viewed under a microscope is dull with uniform purplish color. However, peacock feathers glimmers under normal lighting and if you change your perspective the tint of color changes. The feather is said to be iridescent. Explain the cause of this iridescence? [5 marks]

(b) A transmitting antenna for radio waves of wavelength 5 m is located on a cliff overlooking an open sea. The cliff is 200 m above sea level.

(i) An aeroplane flying just above the surface of the water 20 km away cannot receive signals from the antenna and thus does not reflect echo signals back to the transmitting site. Explain qualitatively why this is so. [5 marks]

(ii) If the plane is flying at certain attitudes above the water surface, the plane will reflect exceptionally strong echoes. Determine these attitudes. [6 marks]

(c) Fig 2 shows a transmission echelon. It is made of a stack of N glass plates of refractive index n and thickness t, assembled so that each plate projects a distance s beyond the preceding one.

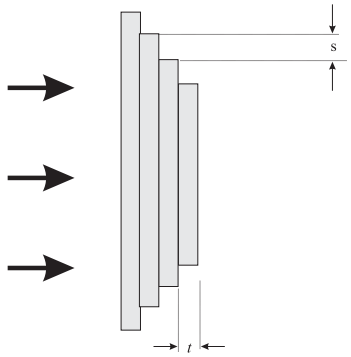

Figure 2: Transmission echelon.

(i) The stack is is illuminated from the left with monochromatic light of wavelength λ. By considering the light coming from each step to the assembly, what is the condition for constructive interference beyond the stack for small angles with respect to the incident beam? [8 marks]

(ii) Determine the order of the interference for $n = 1.5$, $t = 0.5cm$ and $\lambda = 5 \times 10^{-9}$ m? [5 marks]

(iii) Calculate the angular dispersion and resolving power for a stack of 40 plates of this index and thickness, assuming that there is negligible variation in the refractive indices. [6 marks]

3. (a) A singer, holding a note of the right frequency, can shatter a glass if the glassware is of high quality. This cannot be done with one of low quality. Explain why. [5 marks]

(b) A T-shaped structure is formed from two uniform rods, A and B, each of length L and mass m (see figure 3). The mid-point of rod B is attached to one end of rod A. A point mass m moving horizontally at right angles to rod A strikes the end of rod B with an initial velocity V and sticks to it.

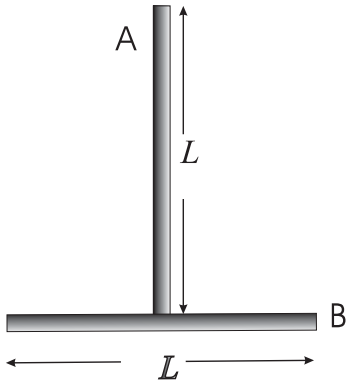

Figure 3: T-shaped structure formed from two rods.

(i) Find the position of point on the T-shaped structure which remains stationary. [5 marks]

(ii) Determine the angular velocity of system immediately after the collision. [5 marks]

(iii) Sketch the positions of the T-structure in subsequent motion. [5 marks]

(iv) Find out the change in kinetic energy of the system as a result of collision. [5 marks]

The T-structure is then suspended from the free end of rod A and allowed to move freely in the plane of the "T". Show that the period of small oscillation is

$$2\pi\sqrt{\frac{17L}{18g}}.$$

[5 marks]

END OF PAPER