Sixteenth Singapore Physics Olympiad

Theoretical Paper

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Organized by

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Instructions to Candidates

1. This is a four-hour test.

2. This paper consists of **four (4)** questions printed on **four (4)** pages.

3. Attempt all questions. Each question carries equal marks.

4. Write your name legibly on the top right hand corner of every answer sheet you submit.

5. Begin each answer on a fresh sheet of paper.

6. Submit **all** your working sheets. No paper, whether used or unused, may be taken out of this examination hall.

7. No books or documents relevant to the test may be brought into the examination hall.

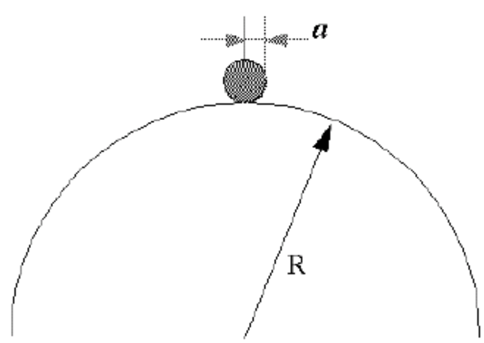
1. (a) A nucleus of mass M_1 and kinetic energy K_1 moves along the *x*-axis and is incident on a stationary nucleus of mass M_2. The collision results in two new nuclei, one of them of mass M_3 and kinetic energy K_3, moving at an angle of ϕ to the *x*-axis and the other one of mass M_4 and kinetic energy K_4, moving at an angle of θ to the *x*-axis. Assuming that the particles are moving at non-relativistic speeds, show that the Q-value of the reaction (difference in energies before and after the reaction) is

$$Q = K_3\left(1 + \frac{M_3}{M_4}\right) - K_1\left(1 - \frac{M_1}{M_4}\right) - \frac{2\sqrt{M_1 K_1 M_3 K_3}}{M_4}\cos\phi$$

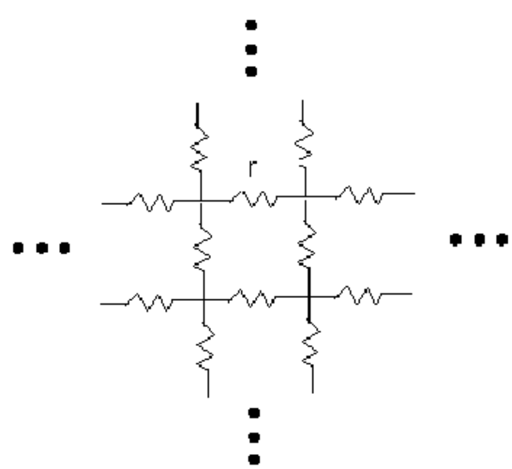
(b) Two wavelengths λ and $\lambda + \Delta\lambda$ (with $\Delta\lambda << \lambda$) are incident on a diffraction grating with slit separation, *d*. Show that the angular separation between the spectral lines in the *m*th order spectrum is

$$\Delta\theta = \frac{\Delta\lambda}{\sqrt{\left(\frac{d}{m}\right)^2 - \lambda^2}}.$$

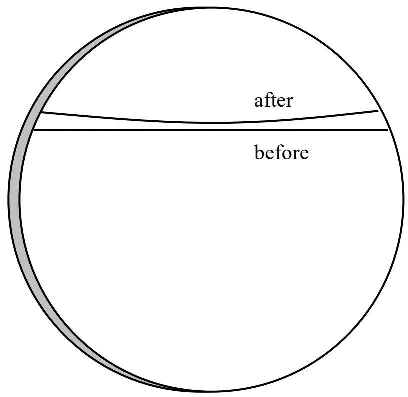
(c) A marble rests on the top of a hemispherical bowl as shown in the figure below. The radii of the bowl and the marble are *R* and *a* respectively. The marble is displaced from rest and it rolls down the bowl without slipping. At what height, in terms of *R* and *a*, will the marble leaves the hemispherical?



(d) Consider the infinite square array of resistors each of resistance *r* as shown in the figure. Find the equivalent resistance between two neighboring point separated by a resistor *r*.



2. (i) Alan has drawn a straight line on a coin, which is made of some homogeneous metal with coefficient of linear thermal expansion α. He is convinced that if the coin is heated up, the line will become curved as shown in the figure:



because some parts of the line are farther from the centre of the coin and therefore will be pushed farther out. Is he correct or wrong? Can you prove it?

Hint: The coefficient of linear thermal expansion α is defined by the relation $l = l_0(1 + \alpha \Delta T)$, where ΔT is the change in temperature and l is the distance between any two points on the object.

(ii) Alan has another similar coin, but now with a hole drilled through its centre. He thinks that if the coin is heated up, the hole must get smaller, since the metal will expand in all directions — inward as well as outward. Is he correct or wrong? Can you prove it?

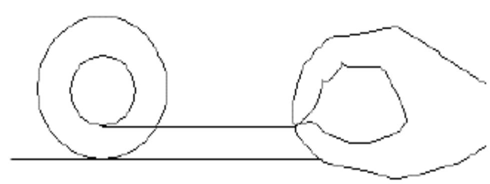
3. A yo-yo is constructed of two brass disks whose thickness b is 8.5 mm and whose radius R is 3.5 cm joined by a short axle whose radius R_0 is 3.2 cm.

(i) Neglecting the moment of inertia of the axle and taking the density of brass to be 8400 kg m^{-3}, what is the moment of inertia about the central axis?

(ii) A string of length 1.1 m and negligible thickness is wound on the axle. What is the linear acceleration of the yo-yo as it rolls down the string from rest?

(iii) Calculate the tension in the string of the yo-yo.

(iv) The yo-yo is then held on level surface. A gentle horizontal pull is exerted on the cord so that the yo-yo rolls without slipping. The yo-yo will roll towards the pull. Explain quantitatively using appropriate equations why this is so.



4. Comets are rocky dust covered with ice. When a comet comes close to the Sun, sublimation of ices in the comet's nucleus creates two tails: a plasma tail made of ionized gas and a dust tail made of solid particles. The dust released by a comet does not continue along the comet's orbit in general because of the radiation pressure from the Sun. The radiation pressure from the Sunlight tends to push the dust particles radially outward from the Sun.

(i) Assuming the intensity of the Sunlight on a dust particle at a distance R from the Sun is I, and the power radiated by the Sun is P, write down a relation between I, P and R. Find also a relation for the force exerted by the radiation on a particle of radius r.

(ii) By considering the gravitational force on a dust particle and taking the density of the dust particle to be ρ, find the radius r of the particle, find the radius r of the particle in terms of P and ρ. Show that the radius is independent of the particle's distance R from the Sun.

(iii) Using the data given below, calculate the value for the radius r of the dust particle given that P is 3.9×10^{26} W.

(iv) The motion of the Halley's comet is an elliptical orbit with an eccentricity of 0.967. The point where the comet makes its closest approach to the Sun is called the perihelion and the farthest distance between a comet and the Sum is the aphelion. Suppose the distance between the perihelion and the aphelion is 34.6 astronomical units (AU), find the distances of closet and farthest approach of the comet. Find also the period of the orbit of Halley's comet around the Sun.

(1 AU = 1.5×10^{11} m)

Data:

$$\rho = 1.0 \times 10^{3}\ \mathbf{kg\,m^{-3}}$$
$$\mathbf{G} = 6.67 \times 10^{-11}\ \mathbf{N\,m^{2}\,kg^{-2}}$$
$$c = 3.0 \times 10^{8}\ \mathbf{m\,s^{-1}}$$
$$M_s = 1.99 \times 10^{30}\ \mathbf{kg}$$

End of Paper