Fifteen Singapore Physics Olympiad

Theoretical Paper: Part II

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Organized by

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Instructions to Candidates

1. This is Part II of a two-part test. This part of the test is a three-hour test.

2. This paper consists of **four (4)** questions and **five (5)** pages.

3. Attempt all questions. Each question carries equal marks.

4. Write your name legibly on the top right hand corner of every answer sheet you submit.

5. Begin each answer on a fresh sheet of paper.

6. Submit **all** your working sheets. No paper, whether used or unused, may be taken out of this examination hall.

7. No books or documents relevant to the test may be brought into the examination hall.

Theoretical Paper: Part II

1 (a) A block *P* of mass *m* kg slides along a horizontal frictionless track with speed *v* ms^{-1}. The block collides with another stationary block *Q* of mass M kg that has a light spring (of spring constant *k*) attached to it as shown in the figure 1. As the blocks collide, block *P* slows down while block *Q* speeds up and the spring compresses. Stating any assumptions, find the maximum compression of the spring.

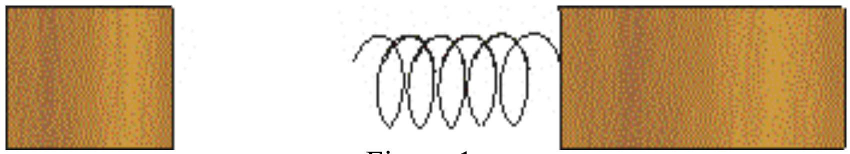


Figure 1

(b) A solid sphere with radius *r* and mass *m* rotates inside a hollow sphere of radius *R* as shown figure 2. Calculate the period of small oscillations about the equilibrium. You may take the moment of inertia of the solid sphere as 2/5 mr^2.

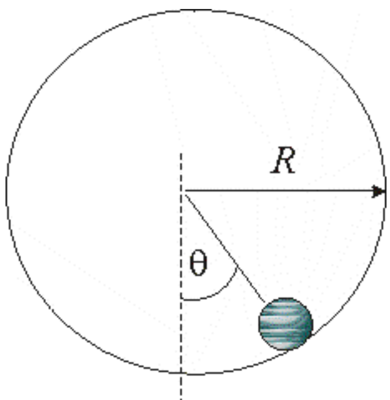


Figure 2

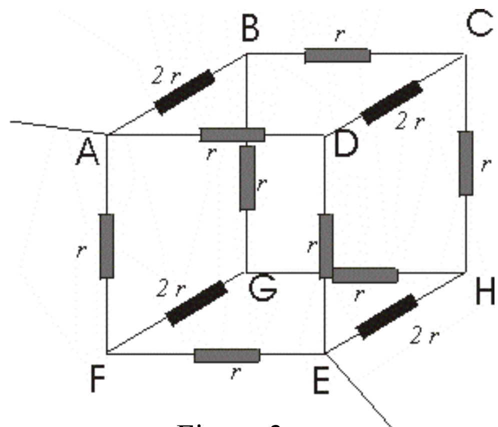


Figure 3

(c) ABCDEFGH is a cuboid with resistances *2r* along the edges AB, CD, FG and EH and resistance r along the rest of the edges. Find the equivalent resistance of the circuit between the points *A* and *E* as shown in figure 3

(d) Find the energy and wavelength of a photon that can impart a maximum energy of 60 keV to a free electron.

(e) A lens is coated with a thin film with a refractive index of 1.2 in order to reduce the reflection from the surface at a wavelength of 5000Å. The glass of the lens has refractive index 1.4. What is the minimum thickness of the coating that will minimize the intensity of the reflected light? Explain why the intensity of the reflected light is small but not zero.

2. (a) The pressure in a balloon depends on its radius. Describe carefully an experiment in which you can measure the pressure in a balloon as a function of radius. State all necessary apparatus and precautions needed for the experiment. How does the pressure in the balloon vary with the radius when it is first blown and then allowed to deflate? Explain your reasoning.

(b) An object falling in a viscous liquid will eventually travels with constant terminal speed. Given that the viscous force, *F*, on a cylinder of radius *r* and density ρ is

$$F = 6 \pi k \eta r^n v$$

where *k* is some geometrical factor associated with the object, η is the coefficient of viscosity, *v* is the velocity of the object, show that the terminal velocity is given by

$$v_T = C r^{3-m} (\rho - \rho')$$

where C is a constant and ρ' is the density of the liquid. Design an experiment using ordinary laboratory apparatus (stating clearly all the items used) to determine the value of the exponent *m*.

3. Phase contrast microscopy can be used to improve the image of transparent biological microbes. The instrument essentially consists of a lens of focal lens f with diameter D as shown in figure 4. A laser beam of wavelength λ illuminates the entire aperture of the lens. The laser beam is parallel to the axis of the lens.

(a) If a transmission grating of line separation d is placed at A, describe the pattern on a screen placed at B.

(b) Suppose you wish to photograph a microbe placed at A (in placed of the grating) and the film is placed on the screen C, how far is C from the lens?

(c) A phase plate is used to improve the microscope. Light goes through the circular phase plate within a distance x from the center suffers a $\pi/2$ phase lag. The plate is placed at B, centered on the optical axis of the lens, with its surface normal to the optical axis. Suppose the refractive index of the microbe differs only slightly from the surrounding medium so that light which passes through the microbe is slightly phase shifted. Compare the image with the addition of the phase plate to the image without the phase plate. How large should x be? If the center of the phase plate is opaque, how is the image changed?

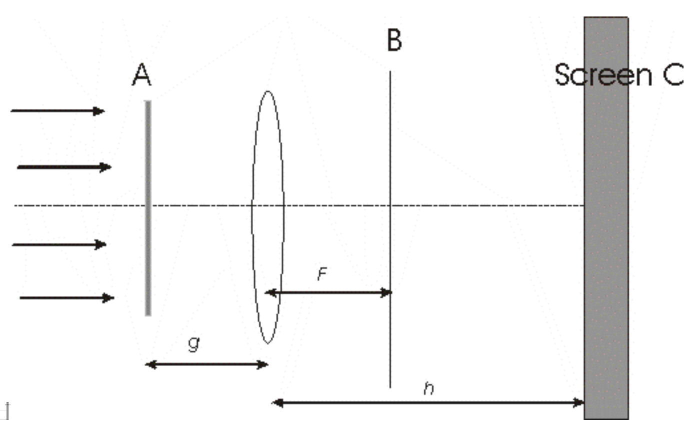


Figure 4

4. (a) The interior of the Earth can roughly be modelled as follows: it consists of a molten core 3440 km in radius, a mantle surrounding the core with thickness 2900 km, and a rocky crust with thickness 30 km. Heat is generated from within the core due to the decay of radioactive elements, and conducts upwards to the surface through the mantle and crust.

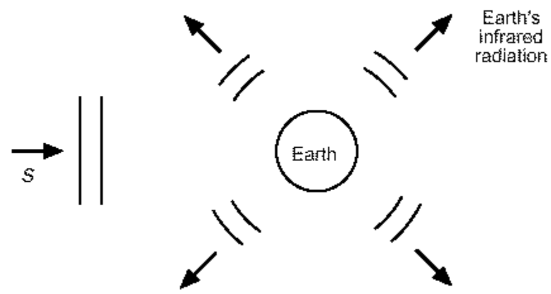
Suppose the core generates 4×10^{13} W of heat and its outer temperature is 3700 °C. If the thermal conductivity of the mantle and the crust is 157 W/(m·°C) and 2.35 W/(m·°C) respectively, what is the resulting temperature at the surface of the Earth assuming there are no other forms of heat gain or loss?

Hint: The rate at which heat flows by conduction through a slab of cross-sectional area *A* and infinitesimal thickness *dx* is

$$H = -kA\frac{dT}{dx}$$

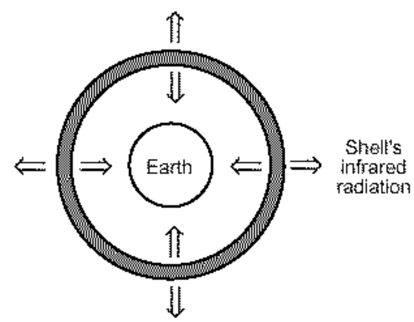
where *dT* is the temperature difference and *k* is the thermal conductivity.

(b) Of course, heat also reaches the Earth's surface from the Sun. The Sun delivers radiation to the Earth at a rate of 1400 W/m^2, measured at right angles to the Sun's rays. However, we must allow for the fact that 30% of the solar radiation is reflected back out to space without being absorbed.



Now, the Earth's surface also radiates infrared radiation back into space. Assuming it radiates like a blackbody, what is the average temperature of the Earth's surface? [Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}$ W/(m^2 K^4)]

(c) A more realistic model takes into account the greenhouse warming effect of the atmosphere. This is done by treating the atmosphere as a spherical shell enclosing the Earth. It is transparent to incoming (visible) solar radiation but absorbs some of the outgoing infrared radiation from the Earth's surface. It is thus heated to a certain temperature and radiates its own infrared radiation, from both the inner and outer surfaces.



Assume that the shell absorbs 75% of the outgoing infrared radiation, and that its (inner or outer) surface area is the same as that of the Earth's surface. Hence calculate the new value for the temperature of the Earth's surface. This value includes the greenhouse warming!