Singapore Junior Physics Olympiad 2024

16 July 2024

1. A smooth hollow cone of half-angle θ is inverted with its axis vertical. A particle of mass m rolls around in a horizontal circle within the inner surface of the cone. What is the magnitude of the normal force N exerted by the cone on the particle?



- A. $mg \cos \theta$ B. $mg \sin \theta$ C. $\frac{mg}{\cos \theta}$ D. $\frac{mg}{\sin \theta}$ E. $mg \tan \theta$
- 2. Chris is seated on a northbound train when he sees a southbound train approaching him with speed u. At the same time, he sees a tree moving away from him at speed v. At what speed does a passenger seated on the southbound train observe this tree to be moving?
 - A. u + vB. u - vC. u - 2vD. 2u + vE. 2u - v
- 3. A particle travels in a straight path with constant acceleration. The particle has velocity u at the start of its path and velocity v at the end of its path. What is the velocity of the particle at the midpoint of its path?

A.
$$\frac{1}{2}(v-u)$$

B.
$$\frac{1}{2}(u+v)$$

C.
$$\sqrt{uv}$$

D.
$$\sqrt{u^2+v^2}$$

E.
$$\sqrt{\frac{1}{2}(u^2+v^2)}$$

4. *n* identical blocks (where n > 2) are placed side-by-side in a horizontal row on frictionless ground. The combined row of blocks is pushed from the left with a force *F*. What is the force between the two rightmost blocks?

A.
$$F$$

B. nF
C. $\frac{F}{n}$
D. $\frac{F}{n-1}$
E. $\frac{(n-1)F}{n}$

The following information applies to both questions 5 and 6.

A uniform chain of length L is placed on a table, with a length $\frac{L}{4}$ hanging over the edge of the table. When the chain is released from rest, it begins sliding off the table.



5. Suppose that the tabletop is frictionless. What is the initial acceleration of the chain?



6. Suppose now that the tabletop has friction. Given that the chain still slides off the table upon release, what is the maximum possible coefficient of static friction of the tabletop?



7. In a "circular pendulum", a mass m is suspended by a string of length ℓ and travels in a circular path at constant speed.



Alice and Bob each propose a free-body diagram for the mass, as shown below.



Whose free-body diagram is correct, and why?

- A. Alice, as the mass must be in equilibrium.
- B. Alice, as the centripetal force is a fictitious force that must be included.
- C. Bob, as tension and weight are the only forces on the mass.
- D. Bob, as tension and weight form an action-reaction pair.
- E. Both are valid diagrams depending on the frame of reference chosen.
- 8. On board a shaky bus, Galen observes a glass of water and notices its surface to be tilted towards the right by an angle $\theta = 35^{\circ}$. What is the acceleration of the bus, and in which direction?



- A. 5.6 m s⁻² towards the left
- B. 5.6 m s⁻² towards the right
- C. 6.9 m $\rm s^{-2}$ towards the left
- D. 6.9 m s⁻² towards the right
- E. 9.8 m $\rm s^{-2}$ downwards

9. A tired delivery worker has a cube-shaped box of mass M and side length L on the ground, beside a step of height $\frac{1}{3}L$. In an attempt to bring the box up the step, he applies a horizontal force F on the box. What is the smallest value of F required for him to succeed?



- 10. A ball is dropped from rest onto an inclined slope. Assuming there are no energy losses, which of the following statements are true about the ball's motion as it bounces down the ramp?
 - I. The time interval between each bounce decreases over time.
 - II. The distance between each bounce, $d_{\rm II}$, increases over time.
 - III. The maximum distance reached by the ball from the ramp between each bounce, $d_{\rm III}$, remains constant over time.
 - IV. The angle of the ball's trajectory from the ramp at each bounce, $\theta_{\rm IV}$, remains constant over time.



- A. I and II only
- B. II and III only
- C. III and IV only
- D. I, II and III only
- E. I, III and IV only

11. Paul fires an arrow at initial speed u towards a target a distance s away from him. He unfortunately makes the mistake of aiming directly at the target, and as a result, his arrow misses the target by a vertical distance d. What is d?



12. Bobo and Nana are initially at rest on icy frictionless ground. Bobo has mass M_1 and Nana has mass M_2 . Bobo grabs a snowball of mass m and throws it at Nana, who catches it. Bobo and Nana end up moving apart at speeds v_1 and v_2 respectively. What is the ratio $\frac{v_2}{v_1}$?

A.
$$\frac{M_1}{M_2}$$

B.
$$\frac{M_2}{M_1}$$

C.
$$\frac{M_1}{M_2 + m}$$

D.
$$\frac{M_2}{M_1 + m}$$

E.
$$\frac{M_1 + m}{M_2 + m}$$

13. The frame of an umbrella is constructed by attaching 6 uniform rods, each of mass m = 0.05 kg and length l = 1.0 m, to a common centre. The frame makes an angle $\alpha = 80^{\circ}$ to the horizontal when the umbrella is closed, and an angle $\beta = 20^{\circ}$ when the umbrella is open, as drawn below. (Note: Not all 6 rods are drawn due to perspective.)



How much work needs to be done to open the umbrella? Assume that the umbrella's central axis is kept vertical and stationary, and that the mass of its fabric is negligible compared to the mass of its frame.

- A. 0.95 JB. 1.1 JC. 1.9 J
- D. 2.3 J
- E. 2.9 J
- 14. Three identical point masses m are connected by strings of length ℓ in the vertical plane, as shown below. The two top masses are constrained to move horizontally along a frictionless track. If the system is released from rest at an initial angle $\theta = 0$ from the horizontal, find the relative velocity of the two top masses just before they collide with each other.



A. 0 B. $\sqrt{g\ell}$ C. $\sqrt{2g\ell}$ D. $\sqrt{3g\ell}$ E. $\sqrt{4g\ell}$ 15. A mass m is suspended at rest by an ideal spring of force constant k. At this position, an extra mass is gently added so that the total mass is now M, before it is released from rest. The spring-mass system subsequently oscillates, but eventually settles to a rest due to damping by resistive forces. How much work is done in total by these resistive forces?



16. Brian wants to bridge two cliffs using a plank of length L. The plank itself is essentially massless, but there is a mass m attached to its front and a mass 4m attached to its back. The bridge is formed by pushing the plank horizontally across the gap, until the full gap is covered by the plank. What is the maximum gap D between the cliffs that can be bridged in this manner?



17. A horizontal turntable of radius R rotates with constant angular velocity ω . An ant, positioned a distance $\frac{3R}{5}$ from the centre, initially rotates along with the turntable. All of a sudden, it decides to jump vertically (relative to itself). What is the minimum duration of its jump such that it escapes the turntable?



A.
$$\frac{2}{5\omega}$$

B. $\frac{3}{5\omega}$
C. $\frac{4}{5\omega}$
D. $\frac{4}{3\omega}$
E. $\frac{5}{3\omega}$

18. A car drives at a constant speed along a horizontal road of width L that has a sharp 90° bend as shown in the figure. The coefficient of static friction between the car and the road is μ . What is the maximum speed at which the car can negotiate the bend without slipping? Neglect the size of the car.



19. A mass is suspended from a light inextensible string of length l = 100 cm. The top end of the string is driven to perform vertical oscillations with amplitude A and period T = 0.25 s. It is observed that the mass oscillates identically to the top end of the string. What is the maximum possible amplitude A?



A. 1.6 cm

- B. 9.8 cm
- C. 31 cm
- D. 39 cm
- E. 61 cm
- 20. A uniform solid disc rests flat on a frictionless table. A brief tangential impulse is applied on the leftmost point of the disc. What is the resulting ratio of speeds of the leftmost and rightmost points of the disc, $\frac{|v_{\text{left}}|}{|v_{\text{right}}|}$?

(The moment of inertia of a uniform solid disc of mass M and radius R about its central axis is given by $I = \frac{1}{2}MR^2$.)



- A. 1 B. $\frac{5}{3}$ C. 2 D. 3
- E. 4

21. A uniform solid sphere is placed at the top of a valley and released from rest. It rolls down, and upon reaching the end of the valley, a height h below the top of the valley, it rises upward into the air. Throughout its motion, the sphere never experiences any energy dissipation and never slips. Yet, Chris claims that the sphere will never return to its original height, falling short by a distance d. Is Chris right, and if so, what is d?

(The moment of inertia of a uniform solid sphere of mass M and radius R about its central axis is given by $I = \frac{2}{5}MR^2$.)



- A. Chris is wrong, as the sphere should return to its original height.
- B. Chris is right, but there is not enough information to calculate d.
- C. Chris is right, $d = \frac{1}{10}h$. D. Chris is right, $d = \frac{2}{7}h$. E. Chris is right, $d = \frac{2}{5}h$.
- 22. Two satellites X and Y orbit the same planet and have equal masses. The orbit of satellite X is circular, whereas the orbit of satellite Y is elliptical. The major axis of satellite Y's orbit has the same length as the diameter d of satellite X's orbit. Which of the following statements are true?
 - I. The period of the orbit is equal for both satellites.
 - II. The total energy of the orbit is equal for both satellites.
 - III. The angular momentum of the orbit (with respect to the planet) is equal for both satellites.
 - IV. The speeds of Satellite Y at points 1, 2 and 3 of its orbit (as shown below) are related by $v_1 < v_2 < v_3$.



- A. III onlyB. I and II onlyC. I and IV onlyD. III and IV onlyE. II, III and IV only
- 23. In an experiment gone very wrong, Shuhao accidentally created a clone of Earth with the same mass and radius R as our Earth. The two Earths are initially stationary with a separation of 8R between their centres. As they attract each other due to gravity, they eventually crash into each other. What is the velocity of each Earth at impact?



24. An L-shaped pipe is placed into a river flowing at speed v, with the open end facing the current head-on. The water in the pipe is observed to rise to a height h = 0.50 m above the water surface of the river. Find v.



25. A wooden cube of density ρ_1 and side length ℓ floats in a fluid of density ρ_2 and undergoes a periodic vertical bobbing motion. Assuming the cube remains upright at all times, what is the period of the bobbing?

A.
$$\sqrt{\frac{\rho_1 \ell}{\rho_2 g}}$$

B. $\sqrt{\frac{\rho_1 g}{\rho_2 \ell}}$
C. $2\pi \sqrt{\frac{l}{g}}$
D. $2\pi \sqrt{\frac{\rho_1 \ell}{\rho_2 g}}$
E. $2\pi \sqrt{\frac{\rho_2 \ell}{\rho_1 g}}$

26. A ray of light enters a spherical droplet. Its resulting trajectory traces the shape of a regular hexagon. What is the minimum possible refractive index of this droplet?



A. 2
B.
$$\sqrt{3}$$

C. $\frac{2}{\sqrt{3}}$
D. $\frac{\sqrt{3}}{2}$
E. $\frac{3}{2}$

27. A man of height h = 1.6 m stands facing a plane mirror hung vertically. The bottom of the mirror is at vertical distance $y_1 = 0.9$ m above the man's feet, while the top of the mirror is at vertical distance $y_2 = 0.4$ m below the top of the man's head. What percentage of his body is the man able to see in the mirror? Treat the man's eye level to be approximately that of the top of his head.



- A. 9.4%
- B. 19%
- C. 38%
- D. 44%
- E. 63%
- 28. Rays from the Sun are brought through a convex lens with a focal length of f = 20 cm and a circular cross section of diameter d = 10 cm. This produces a bright spot on a screen placed a distance x = 15 cm behind the lens. What is the diameter of this bright spot?
 - A. 2.5 cm
 - B. 5.0 cm
 - C. 6.0 cm
 - D. 7.5 cm
 - E. 10 cm
- 29. Consider a cube of side length a. 7 identical charges +q and one additional charge -q are placed on the vertices of the cube, as shown below. Find the electric field at the centre of the cube.



A. 0 B. $\frac{1}{4\pi\varepsilon_0}\frac{2q}{a^2}$ C. $\frac{1}{4\pi\varepsilon_0}\frac{4q}{a^2}$ D. $\frac{1}{4\pi\varepsilon_0}\frac{4q}{3a^2}$ E. $\frac{1}{4\pi\varepsilon_0}\frac{8q}{3a^2}$ 30. Consider again a cube of side length a. This time, a charge +q is spread uniformly across one of the insulating faces of the cube. A charge +Q is now placed at the centre of the cube. Find the electric force exerted on the charge +Q.



A.
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a^2}$$

B.
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{6a^2}$$

C.
$$\frac{1}{4\pi\varepsilon_0}\frac{2Qq}{3a^2}$$

D.
$$\frac{1}{\varepsilon_0}\frac{Qq}{a^2}$$

E.
$$\frac{1}{\varepsilon_0}\frac{Qq}{6a^2}$$

31. A conducting sphere of charge +Q and radius *a* is placed in the centre of a spherical conducting shell of charge -2Q and radius *b*. What is the minimum work required to bring a small charge +q from the outer shell to the surface of the inner sphere?



A.
$$\frac{Qq}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right)$$

B.
$$\frac{Qq}{4\pi\varepsilon_0} \left(\frac{1}{b} - \frac{1}{a}\right)$$

C.
$$\frac{Qq}{4\pi\varepsilon_0} \frac{1}{a}$$

D.
$$\frac{Qq}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{2}{b}\right)$$

E.
$$\frac{Qq}{4\pi\varepsilon_0} \left(\frac{1}{a} + \frac{2}{b}\right)$$

32. Find the power dissipated by the 3 Ω resistor in the circuit below.



B. 5.3 W
C. 7.5 W
D. 8.3 W
E. 16 W

A. 2.7 W

33. A tetrahedron is made out of 6 identical resistors of resistance R forming each of its edges, as shown below. What is the effective resistance between two adjacent vertices of the tetrahedron?





34. Three initially uncharged capacitors are connected with a battery as shown in the circuit below. The switch is first set to position A, and after a long time, the switch is then set to position B. Find the final charge on the 40 μ F capacitor.



- A. 220 μC
 B. 330 μC
 C. 440 μC
 D. 530 μC
- E. 670 μ C
- 35. Two horizontal wires A and B carry currents in perpendicular directions, with the top view shown in the diagram below. Wire B is fixed in place at a small distance below wire A, and wire A is free to rotate about a pivot at the point directly above wire B. What happens to wire A when it is released from rest?



- A. It will rotate counter-clockwise.
- B. It will rotate clockwise.
- C. It will rotate, causing point X (marked above) to move out of the page.
- D. It will rotate, causing point X (marked above) to move into the page.
- E. It will not rotate as there is no net torque.
- 36. An electron is imparted an initial velocity v_0 in a region of uniform magnetic field *B* directed into the page, as shown below. What is the subsequent motion of the electron?



- A. It will move in a straight line.
- B. It will move in a clockwise circle.
- C. It will move in a counter-clockwise circle.
- D. It will move in a helix towards the top.
- E. It will move in a helix towards the right.

37. Two identical thin, wide and parallel square metal plates each carry a uniform current I in the same direction along their length. What is the force that one plate exerts on the other?



- A. 0 B. $\mu_0 I^2$ C. $\frac{1}{2}\mu_0 I^2$ D. $\frac{1}{\pi}\mu_0 I^2$ E. $\frac{1}{2\pi}\mu_0 I^2$
- 38. A metal disc of radius r is driven to rotate with constant angular velocity ω about its axis. A uniform magnetic field B is directed perpendicular to the disc's surface and covers its entire area. What is the induced voltage between the centre of the disc and its edge?



- A. 0 B. $\pi r \omega^2 B$ C. $\pi r^2 \omega B$ D. $\frac{1}{2} r^2 \omega B$ E. $r^2 \omega B$
- 39. Shaun strings two beads on a light uniform thread, one of mass m_1 fixed at the middle and another of mass m_2 fixed at the bottom, and suspends the thread from the top. He then "pokes" the top, creating a tiny transverse disturbance that sends a wave downward. As the wave moves through the thread, its wavelength changes from λ_1 to λ_2 . What is the ratio $\frac{\lambda_1}{\lambda_2}$?



A. 1
B.
$$\sqrt{\frac{m_1}{m_2}}$$

C. $\sqrt{\frac{m_1}{m_1 + m_2}}$
D. $\sqrt{\frac{m_1 + m_2}{m_1}}$
E. $\sqrt{\frac{m_1 + m_2}{m_2}}$

40. The displacement y of a transverse wave on a string of length L can be written as a function of position $x \ (0 \le x \le L)$ and time t:

$$y(x,t) = A\sin\left(\frac{\pi x}{L}\right)\cos\omega t + \frac{1}{3}A\sin\left(\frac{2\pi x}{L}\right)\sin\omega' t$$

Which of the following statements are true?

- I. $\omega' = 2\omega$.
- II. This string is fixed on one end only.
- III. The first term in the above expression for y(x,t) represents the first harmonic mode, while the second term represents the second harmonic mode.
- IV. The ratio of energies carried by the two modes is 3:1.
 - A. I only
 - B. I and III only
 - C. II and IV only
 - D. I, III and IV only
 - E. I, II, III and IV
- 41. Two sources are set in phase and produce beams of plane-polarised light that have equal amplitudes and are directed towards each other. A detector is placed at the midpoint of the two sources. If the light from both sources are set to have parallel polarisations, the detector measures a resulting amplitude of A. If the light from both sources are instead set to have perpendicular polarisations, what is the resulting amplitude measured by the detector?
 - A. 0 B. A C. $\frac{A}{2}$ D. $\frac{A}{\sqrt{2}}$ E. $\sqrt{2}A$

42. Consider a variation of Young's double slit experiment where the light entering the two slits have different intensities I_1 and I_2 , with $I_1 > I_2$. Let the intensities of the resulting bright and dark fringes be I'_1 and I'_2 respectively. What is the ratio $\frac{I'_1}{I'_2}$?

A.
$$\frac{I_1}{I_2}$$

B. $\sqrt{\frac{I_1}{I_2}}$
C. $\frac{I_1 + I_2}{I_1 - I_2}$
D. $\sqrt{\frac{I_1^2 + I_2^2}{I_1^2 - I_2^2}}$
E. $\left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2$

43. Consider now an ideal diffraction grating with slit spacing d. Light of wavelength λ enters the grating normally. On a wide screen placed far away, only a total of 5 peaks of bright fringes can be observed across both sides of the entire screen. What is the maximum possible value of λ ?

A.
$$d$$

B. $\frac{1}{2}d$
C. $\frac{1}{3}d$
D. $\frac{1}{5}d$
E. $\frac{1}{6}d$

- 44. Paul has a glass of water that contains a mass $m_w = 400$ g of water at temperature $T_0 = 30^{\circ}$ C. He tries to cool it down by adding a mass $m_i = 100$ g of ice cubes. Assuming that no heat is exchanged with surroundings, what is the final temperature of Paul's water?
 - A. 0° C, the ice does not melt completely.
 - B. 4.0°CC. 8.1°C
 - D. 10.1°C
 - E. 15.9°C
- 45. Three metal slabs are placed in thermal contact between a hot reservoir of temperature $T_{\text{hot}} = 100^{\circ}\text{C}$ and a cold reservoir of temperature $T_{\text{cold}} = 20^{\circ}\text{C}$. The slabs have thermal conductivity $k_1 = 200 \text{ W m}^{-1} \text{ K}^{-1}$, $k_2 = 400 \text{ W m}^{-1} \text{ K}^{-1}$ and $k_3 = 600 \text{ W m}^{-1} \text{ K}^{-1}$ respectively. The slabs have the same horizontal length and thickness, but slabs 2 and 3 each have half the vertical length of sheet 1. Neglecting any temperature variation in the vertical direction, find the equilibrium temperature at the centre where all three slabs meet.



- A. 40°C
 B. 50°C
 C. 60°C
 D. 70°C
 E. 80°C
- 46. A bimetallic strip is made up of two metals with linear expansion coefficients α_1 and α_2 , with $\alpha_1 > \alpha_2$. Each metal strip is initially straight with length L_0 and thickness d, where $d \ll L_0$. The bimetallic strip is heated uniformly such that its temperature rises by ΔT , causing it to bend with an angle of curvature θ as shown below. Find θ .



A.
$$\frac{L_0}{d}(\alpha_1 - \alpha_2)\Delta T$$

B.
$$\frac{d}{L_0}(\alpha_1 - \alpha_2)\Delta T$$

C.
$$\frac{d}{L_0}(\alpha_1 + \alpha_2)\Delta T$$

D.
$$d(1 + \alpha_1 + \alpha_2)\Delta T$$

E.
$$L_0(1 + \alpha_1 + \alpha_2)\Delta T$$

47. A planet of radius r and surface temperature T_p orbits a sun of radius R and surface temperature T_s . The distance between the planet and the sun is d, where $d \gg r, R$. Assuming that the planet and the sun are both ideal blackbodies, find the ratio $\frac{T_p}{T_s}$ at equilibrium.

A.
$$\sqrt{\frac{R}{r}}$$

B. $\sqrt{\frac{R}{d}}$
C. $\sqrt{\frac{R}{2d}}$
D. $\sqrt[4]{\frac{R^2}{2d^2}}$
E. $\sqrt[4]{\frac{R^2}{2r^2}}$

48. Two vessels of equal volumes are connected together by a tube and contain a total of N ideal gas molecules. Initially, both vessels have equal temperatures. The temperatures of the vessels are then changed and maintained at $T_1 = 250$ K and $T_2 = 350$ K respectively. As a result, the molecules redistribute themselves across the two vessels. What is the net number of molecules that are eventually transferred between the vessels?

A.
$$\frac{1}{12}N$$

B. $\frac{2}{7}N$
C. $\frac{2}{5}N$
D. $\frac{5}{12}N$
E. $\frac{5}{7}N$

49. An ideal gas undergoes a heat cycle that is represented by the following T - V diagram.



In one cycle, ______ of work is done ______

- A. $2nRT_0$, by the gas.
- B. $2nRT_0$, on the gas.
- C. $nRT_0 \ln 3$, by the gas.
- D. $nRT_0 \ln 3$, on the gas.
- E. $\frac{3}{2}nRT_0$, on the gas.
- 50. Zeyuan has a balloon with perfectly elastic and insulating walls. He first fills the balloon at sea level with air at temperature $T_0 = 300$ K and seals it airtight. He then immerses the balloon underwater to a depth of h = 10 m. Find the final temperature of the air in the balloon. Assume that air is diatomic.
 - A. 247 K
 - B. 256 K
 - C. 300 K
 - D. 349 K
 - E. 364 K