MS2-1

The Physics of a Microwave Oven – Marking Scheme

Part A: The structure and operation of a magnetron

A.1	realized $f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}}$	0.1pts
	found $L = 0.6\pi\mu_0 R^2/h$	0.2 pts
	final result	0.1 pts
A.2	found $\vec{u}_D = -(E_0/B_0)\hat{x}$	0.1 pts
	found $r = mu'/eB_0$ (or equivalent, e.g., using ODE)	0.2 pts
	For $\vec{u}(0) = \left(\frac{3E_0}{B_0}\right)\hat{x}$:	
	found $u' = \frac{4E_0}{B_0}$ (or equivalent, e.g., using ODE)	0.1 pts
	found $r = 4mE_0/eB_0^2$ (or equivalent, e.g., using ODE)	0.1 pts
	For $\vec{u}(0) = -(3E_0/B_0)\hat{x}$:	
	found $u' = 2E_0/B_0$ (or equivalent, e.g., using ODE)	0.1 pts
	found $r = 2mE_0/eB_0^2$ (or equivalent, e.g., using ODE)	0.1 pts
	For each graph:	
	correct direction of drift (towards $-\hat{x}$)	0.1 pts
	correct trajectory pitch ($2\pi mE_0 / eB_0^2$)	0.1 pts
	correct y range	0.1 pts
	correct graph shape	0.1 pts
A.3	realized $u' < v_{max}$, hence $r < \frac{mv_{max}}{eB_0}$	0.2 pts
	connection between velocity and energy	0.1 pts
	final result	0.1 pts
A.4	drifts in all five points are correct	1.2 pts
	only four are correct	0.6 pts
	less than four are correct	0.0 pts
	all five points are reversed	0.6 pts

A.5	all six pairs are correct	1.2 pts
	only five pairs are correct	0.6 pts
	less than five pairs are correct	0.0 pts
	all six pairs are reversed	0.6 pts
A.6	direction of rotation	0.1 pts
	four spokes	0.2 pts
	spokes are symmetric under $90°$ rotation	0.1 pts
	in half period the spoke rotates by $45°$	0.2 pts
	correct result of angular velocity	0.2 pts
A.7	realized $u_D = \dfrac{V_0}{(b-a)B}$	0.3 pts
	using $u_D = \pi f (b+a)/4$	0.5 pts
	correct result	0.3 pts

Part B: The interaction of microwave radiation with water molecules

B.1	found $\tau(t) = p_0 \lvert E(t) sin\theta(t) \rvert$	0.2 pts
	found that $H_i(t) = -p_0 E(t) \sin\theta(t)\, \dot{\theta}(t)$	0.3 pts
B.2	realized polarization is along x axis	0.1 pts
	$\langle H(t) \rangle = \left\langle E_0 \sin(\omega_f t) \dfrac{d}{dt} \left(\beta \varepsilon_0 E_0 \sin(\omega_f t - \delta) \right) \right\rangle$	0.1 pts
	used correct trigonometric averaging	0.1 pts
	final result	0.2 pts
B.3	found the average electromagnetic energy density $\dfrac{\varepsilon_0 \varepsilon_r}{2} {E_0}^2(z)$	0.1 pts
	the energy flux density is $\dfrac{\varepsilon_0 \varepsilon_r}{2} {E_0}^2(z) \cdot \dfrac{c}{\sqrt{\varepsilon_r}}$	0.2 pts
	used result of task B2 to get	0.4 pts

	$I(z+dz) = I(z) - \frac{1}{2}E_0^2(z)\beta\varepsilon_0\omega_f \sin\delta\, dz$	
	obtained $\frac{dI(z)}{dz} = -I(z)\cdot\frac{\beta\omega_f \sin\delta}{c\sqrt{\varepsilon_r}}$	0.2 pts
	final result $I(z) = I(0)\exp\left(-\frac{\beta\omega_f \sin\delta}{c\sqrt{\varepsilon_r}}z\right)$	0.2 pts
B.4	found the average energy density $\frac{\varepsilon_0\varepsilon_r}{2}E_0^2 e^{-k_0 n\tan\delta z}$	0.2 pts
	found $I(z) = I(0)e^{-\frac{\omega_f\sqrt{\varepsilon_r}\tan\delta}{c}z}$	0.2 pts
	final result $\beta = \varepsilon_r$	0.2 pts
B.5	used $e^{-\frac{\omega_f\sqrt{\varepsilon_r}\tan\delta}{c}z} = e^{-\ln 2}$	0.1 pts
	found correct numerical value of z	0.2 pts
	for water penetration depth: increases with increasing temperature	0.2 pts
	for soup penetration depth: decreases with increasing temperature	0.2 pts