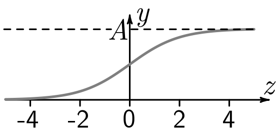
E1 – Deep Learning (10 pts)

Modern ANNs (artificial neural networks) are made of billions of neurons. Each neuron transforms its input(s) $x_1, x_2, ..., x_n$ to an output y. First,

$$z = w_1x_1 + w_2x_2 + \cdots + w_nx_n + b$$

is calculated, with real numbered *weights* w_i and real numbered *bias* b. Then an *activation function* is applied to z to produce the final output $y(x_1, x_2, \ldots)$. In the present problem you will investigate a physical model of a neuron with the electric voltages x_1 and x_2 as inputs, with the activation function being $A\sigma(z)$, graphed below, where $\sigma(z) = 1/(1+\exp\ (-z))$ is called sigmoid function.

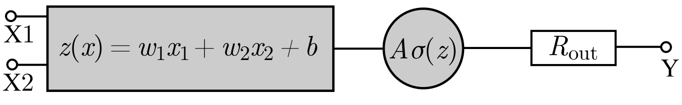


Equipment

Important! Do not switch off any power outlets.

(i) A box containing a voltage source, an electronic circuit that models the neuron, and two potentiometers (the A-potentiometer and the B-potentiometer). The electric terminals on the box are denoted as follows:

1 Two electrically connected GND terminals: the electrical *ground* serving as a common negative terminal for +V, x_1, x_2, and y.
2 +V: the positive terminal of the voltage source.
3 X1 and X2: the positive terminals of the neuron input voltages x_1 and x_2, respectively. **The neuron output behaves unpredictably if either of these terminals has no input voltage.**
4 Y: the positive output terminal. It behaves like a real voltage source, consisting of an ideal voltage source of voltage y and a series output resistor R_{out}, and operates as shown below.



5 A1, A2, A3: terminals of the A-potentiometer.
6 B1, B2, B3: terminals of the B-potentiometer.
7 T: a terminal not to be used in this task.

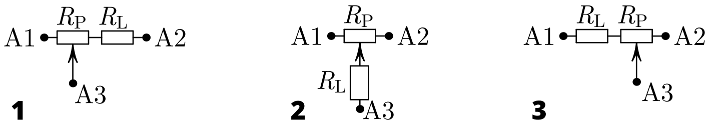
(ii) Digital multimeter with two probe wires.

(iii) Wires with banana connectors. Two or more wires could be connected to the same terminal in the box by using the holes in the banana connectors. Using the banana connectors with the multimeter may form an unstable connection. Use the alligator clamp if needed.

(iv) Graph paper. You can ask for more if needed.

Task 1 (0.5 pts)

Terminals A1, A2, and A3 are connected to the A-potentiometer R_P and an additional load resistor R_L. Which of the schemes below corresponds to the circuit in the box? Determine the resistances R_L and R_P; document the measurements made.



Note The B-potentiometer is connected to terminals B1, B2, B3 in exactly the same way with the same resistances R_L and R_P, within manufacturing tolerances.

Task 2 - (0.5 pts)

Sketch how the terminals have to be connected so that the neuron input voltages can be varied with the widest possible range.

Task 3 - (1.5 pts)

Devise (and document) a strategy allowing you to find the combination of input voltages x_1 and x_2 that maximizes the output voltage y with the least possible number of measurements, irrespectively of with which set of input voltages you start the search. Determine this maximal voltage y_{max} that will be henceforth used as an approximation for the amplitude A, and document your measurements.

Task 4 - (3.5 pts)

Determine the weights w_1, w_2 and the bias b. Describe your measurements and document your data in a table. Estimate w_1, w_2, and b by using a graphical approach.

Training involves optimizing the network weights to achieve desired functionality. This allows ANNs to approximate arbitrary functions. For each of the following tasks you have to approximate a different function of a single input voltage using the given equipment. Make sure that the input and output that you define are clearly marked in your circuits.

Task 5 - (1.5 pts)

Connect the terminal X1 directly to +V. Design a circuit to approximate the function $y_5(x) = A\sigma\left(w_2x/2 + b_5\right)$, where x is the voltage applied to your newly defined input terminal. Determine b_5 theoretically. Implement the circuit, take measurements and verify that your setup works as expected. Validate the value of b_5 from your data.

Task 6 - (2.5 pts)

a Determine the internal series output resistance R_{out} of the Y terminal. (0.5 pts)
b Design and implement a circuit to approximate the function $y_6(x) = A_6 \cdot \sigma(w_2x + b) + B_6$, where $B_6 = 1.48\,\text{V}$. Determine A_6 theoretically. Implement the circuit and verify experimentally that your setup works as expected. Confirm the values of A_6 and B_6 from your data. (2.0 pts)

E2 – Hidden pattern (10 pts)

You are given a flat semi-transparent foil with a micro-pattern printed on its surface that is invisible to the naked eye. The pattern consists of a large number of identical sinusoids with amplitude A, running horizontally with spatial period Λ, and vertically shifted by distance d relative to each other, as schematically shown in Fig. 1. Under a microscope, one can see that the printed pattern is composed of strictly horizontal line segments, each vertically displaced from its neighbours by a constant pitch s, as shown in Fig. 2.

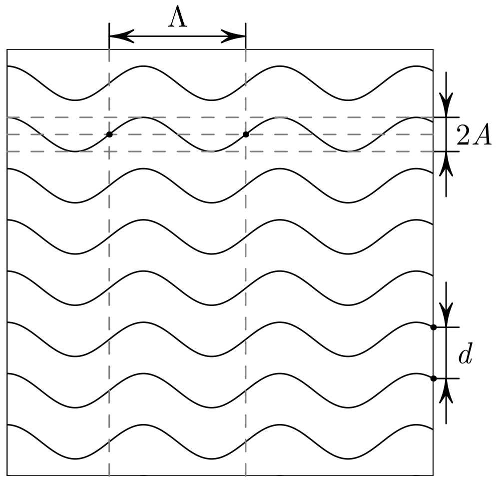


Figure 1: Pattern (not to scale)

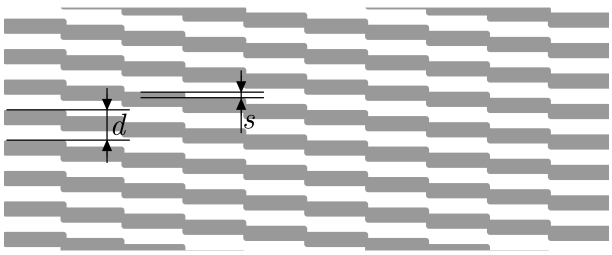


Figure 2: Pattern as seen under microscope

Equipment (see also Fig. 3)

- A Semi-transparent foil with a micro-pattern printed on its surface.
- B Laser diode with wavelength $\lambda = (654 \pm 5)$ nm. The laser diode can be focused to the desired distance by rotating the end cap with a lens inside.

  Warning: Do not completely unscrew the end cap! Inside, there is an oriented lens and a spring. No replacement laser will be given if damaged or disassembled.
- C Two 90-degree L-shaped steel planks serving as stands for the foil and the laser diode. The foil can be fixed to one of the planks using the provided small clips. The laser diode can be mounted to the other plank with a larger colored clip or with the provided rubber band.
- D A sheet of paper with a printed goniometer – a polar coordinate frame with 1-mm radial steps and angular divisions in degrees.

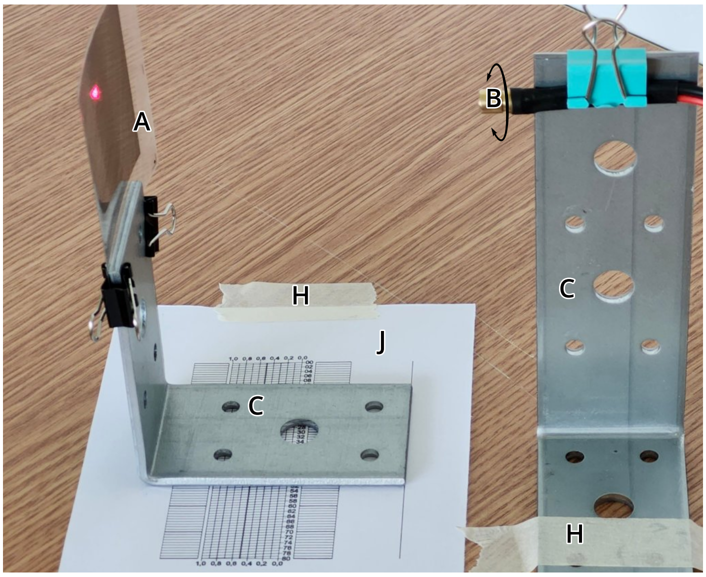


Figure 3: Components A, B, C, H, and J arranged for the experiment.

- E A screen: the large surface of the box containing the experimental materials. Empty the box and place it on the desk with its large surface vertical.
- F Ruler.
- G Measuring tape.
- H Adhesive tape attached to the ruler. Use pieces of the tape to fix the printed goniometer to the screen or to secure components to the table. You can ask for more tape if needed.
- I Millimeter graph paper.
- J An $80\,\text{mm}$ paper measuring scale with diagonal reference lines that allow you to measure fractions of the main scale divisions, accurate to $\pm 0.1\,\text{mm}$.

Hint: In all of your measurements you are free to draw or put marks on the screen.

Important: Assume that the surface of the experimental desk is flat, and the screen is strictly perpendicular to the desk.

Tasks (10.0 pts)

Determine as precisely as possible:

- a The sinusoid period Λ. (2 pts)
- b The vertical offset d of the neighbouring sinusoids (2 pts)
- c The sinusoid amplitude A (3 pts)
- d The step height s (3 pts)

In all of the tasks you are expected to:

1. sketch a setup and/or rationalize a method for measuring the corresponding quantities;
2. report your measurements and calculations in a tabular form;
3. estimate the desired quantities and their uncertainties graphically, whenever reasonable.