T3 - Part A

errors with $\{t_\ell, t_s, a\}$ are physics = dimension!

A1 (0.2)

$$A_{eff} = \pi R^2 \qquad 0.1$$

$$P_0 = (1-a)\,F_s \cdot A_e = (1-a)\,\pi R^2 F_s \qquad 0.1$$

A2 (0.3)

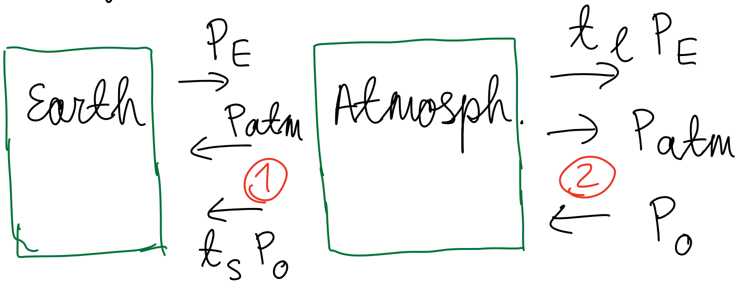
$$P_0 = 4\pi R^2 \sigma T_0^4 \qquad 0.1$$

$$\sigma T_g^4 = \frac{1}{4}(1-a)\,F_s \qquad 0.1$$

$$T_{g_0} = 255\ \text{K} \qquad 0.1$$

(only for correct number!)

A3 (0.7)



$$A = 4\pi R^2$$

Earth: $\quad P_E = P_{atm} + t_s P_0 \qquad 0.2$

Atmosp: $\quad (1-t_s)\,P_0 + (1-t_\ell)\,P_E = 2P_{atm}$

or region (2): $\quad t_\ell P_E + P_{atm} = P_0 \qquad 0.2$

$$\sigma T_g^4 = \frac{1}{4}(1-a)\,F_s \cdot \frac{1+t_s}{1+t_\ell} =$$

or $\quad T_g = T_{g_0} \cdot \left(\frac{1+t_s}{1+t_\ell}\right)^{\frac{1}{4}} \qquad 0.2$

$$T_g = 286\ \text{K} \qquad 0.1$$

(only for correct number)

T3 – Part B

use center of mass

B1 (0.5)

$$m\ddot{X}_A = -m\ddot{X}_B = k(\ell-\ell_0) \quad 0.1+0.1$$

$$\ddot{\ell} = -k\left(\frac{1}{m_A}+\frac{1}{m_B}\right)(\ell-\ell_0) \quad 0.1$$

$$m\ddot{X}_A = -k_{eff}X_A \quad 0.1$$

$$k_{eff} = k\cdot\frac{m_A+m_B}{m_B} \quad 0.2$$

$$\omega^2 = \frac{(m_A+m_B)k}{m_A m_B} \quad 0.2$$

(correct ω without proof → full mark)

B2 (0.2)

$$E_p = \hbar\omega \quad 0.2$$

($h\omega \to 0.1$)

B3 (0.2)

$$f = f_0\left(1+\frac{v}{c}\right) \quad 0.1$$

($f = f_0\sqrt{\frac{1+v/c}{1-v/c}}$ – full mark

$f = f_0\left(1-\frac{v}{c}\right)$ – full mark)

$$f - f_0 = \frac{v}{c}f \quad 0.1$$

B4 (0.2)

$$\int_{-\infty}^{\infty} p(v)\,dv = 1 \quad 0.1$$

$$C^2 = \frac{m}{2\pi kT} \quad 0.1$$

B5 (0.3)

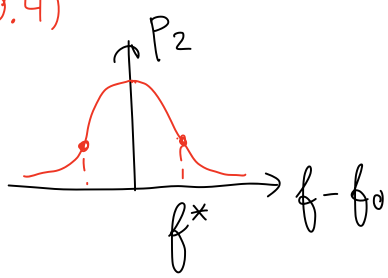
from B3: $v(f) = c\,\frac{f-f_0}{f_0}$ 0.1

$$\Rightarrow P_2(f) \sim P_1(v(f)) \sim \exp\left(-\frac{mc^2}{2kT}\cdot\left(\frac{f-f_0}{f_0}\right)^2\right) \quad 0.2$$

B6 (0.4)

$$P_2 = \frac{P_{2max}}{e} \text{ at } f^* - f_0 = \pm f_0\cdot\sqrt{\frac{2kT}{mc^2}} \quad 0.1$$

(only „+" → full mark)



Graph:

- max at zero 0.1
- symmetry 0.1
- $P_2 \to 0$ on both ends 0.1

T3-Part C

$\boxed{C1}$ (0.3)

force balance 0.1

$$\Delta p = -\rho(z) g \Delta h \quad 0.1$$

$$\frac{dp}{dz} = -\rho(z) g \quad 0.1$$

correct answer 0.3 for C1

$\boxed{C2}$ (0.2)

$$PV = \frac{m}{\mu} RT \quad \text{or} \quad \rho = \frac{P\mu}{RT} \quad 0.1$$

$$\frac{dp}{dz} = -\frac{P(z)\mu g}{RT(z)} \quad 0.1$$

$\boxed{C3}$ (0.2)

$$T = \text{const}: \quad \frac{dp}{p} = -\frac{\mu g}{RT} \cdot dz \quad 0.1$$

$$p(z) = p_0 \exp\left(-\frac{\mu g}{RT} \cdot z\right) \quad 0.1$$

correct answer 0.2 for C3

$\boxed{C4}$ (0.6)

$$p^{1-\gamma} \cdot T^{\gamma} = \text{const} \quad (\text{or } PV^{\gamma} = \text{const, etc}) \quad 0.1$$

$$\frac{dT}{dz} = \frac{\gamma-1}{\gamma} \frac{T}{p} \frac{dP}{dz} = -\frac{\gamma-1}{\gamma} \frac{\mu g}{R} \quad 0.3 \ (\text{any equation})$$

$$\frac{\gamma-1}{\gamma} = \frac{R}{C_p} \Rightarrow \Gamma_a = -\frac{dT}{dz} = \frac{\mu g}{C_p} \quad 0.2$$

$\boxed{C5}$ (1.4)

$$m\ddot{z} = \rho_a(z) g V - mg \quad 0.4$$

$$\ddot{z} = \underbrace{\frac{\rho_a - \rho_p}{\rho_p} g}_{0.2} = \underbrace{\frac{T_p - T_a}{T_a} \cdot g}_{0.2} = \underbrace{\frac{\Gamma - \Gamma_a}{T} g \cdot \Delta z}_{0.2}$$

$$\omega^2 = g \frac{\Gamma_a - \Gamma}{T} = \frac{g}{T}\left(\frac{\mu_{air} g}{C_p} - \Gamma\right) \quad 0.2$$

$$\text{stable, if} \quad \Gamma < \frac{\mu_{air} g}{C_p} \quad 0.2$$

only for correct answers

T3 – Part D

$\boxed{D1}$ (0.5)

$$\Delta S = \frac{\Delta Q}{T} = \frac{L\,\Delta m}{T}$$ 0.1+0.1

$$\Delta V = V_{vap} - V_{wat} \simeq V_{vap}$$ 0.1

$$V_{vap} \simeq \frac{\Delta m \cdot RT}{P\,\mu_{H_2O}} \Rightarrow \frac{dp_s}{dT} = \frac{\mu_{H_2O} \cdot L \cdot P}{RT^2}$$ 0.2

$\boxed{D2}$ (0.2)

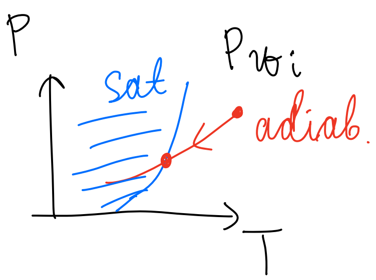
$$\frac{dp}{p} = \frac{\mu L}{R}\,\frac{dT}{T^2}$$ 0.1

$$p_s(T) = p_{s_0} \cdot \exp\left(-\frac{\mu_{H_2O} \cdot L}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right)$$ 0.1

$\boxed{D3}$ (2.0)

$$p_{v_i} = \varphi \cdot \frac{\mu_{air}}{\mu_{H_2O}} \cdot p_i$$ 0.4

$$p(T) = p_{v_i}\left(\frac{T}{T_i}\right)^{\frac{\gamma}{\gamma-1}} \quad \left(\gamma_a = \frac{7}{5}\right)$$ 0.2



$p(T_\ell) = p_s(T_\ell)$ (idea) 0.4

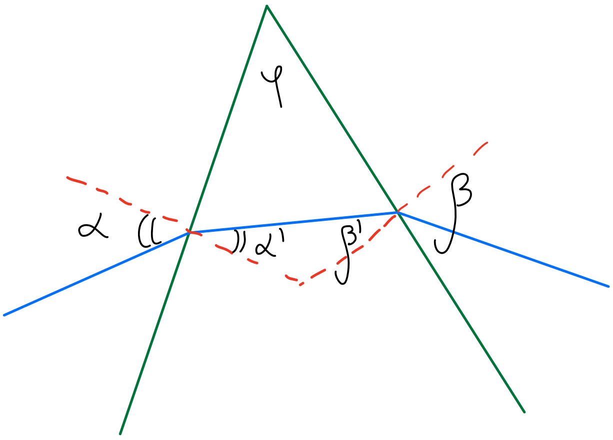
numerical equation 0.4

way to solve transcend. eq. / linear eq. 0.2

$T_\ell = 13.7\,°C \pm 0.3°C$ 0.4 (only for correct number)

Note $\quad p_{v_i} = \varphi\, p_i \Rightarrow$ max 1.2 for D3

T3 - Part E



$\boxed{E1}$ (0.8)

$\sin\alpha = n \cdot \sin\alpha'$ 0.2

$\sin\beta = n \cdot \sin\beta'$ 0.2

$\delta = (\alpha - \alpha') + (\beta - \beta')$ 0.1

(or $\delta = \alpha + \beta - \varphi$)

final formula

$$\delta = \alpha - \varphi + \arcsin\left\{ n \sin\left[\varphi - \arcsin\left(\frac{\sin\alpha}{n}\right)\right]\right\}$$ 0.3

(only for correct formula)

check : $\alpha = \varphi = 60^\circ \Rightarrow \delta = 24{,}721^\circ = 0.43147$ rad

$\boxed{E2}$ (0.6)

Graph: correct shape $\cup$ 0.2

minimum for $\alpha \in [35^\circ; 45^\circ]$ 0.2

$\delta \in [20^\circ; 24^\circ]$ 0.2

$\boxed{E3}$ (0.2)

halo $\Leftrightarrow$ $\delta = \delta_{min}$ 0.1 (no explanation need)

$\delta = 21.8^\circ$ 0.1

Propagating errors

numerical factor
- (−0.2p) here
- full mark later
- 0p for numeric. answer

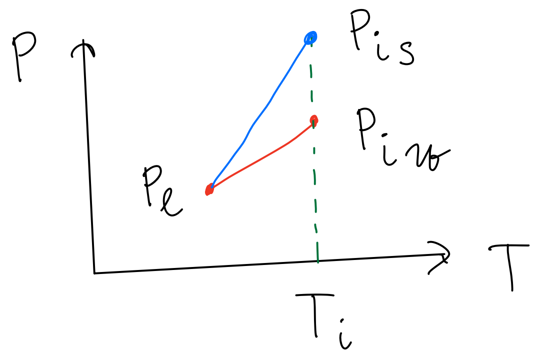
dimensional mistake
- 0p here
- 50% later
- 0p for numeric. answer

Alternative solution for D3

$$P(T_\ell) = P_s(T_\ell)$$

$$|\Delta T| = T_\ell - T_i \ll T_i \Rightarrow$$

up to 1st order in $\frac{\Delta T}{T_i}$



for adiabatic: $(\Delta T < 0)$

$$\Delta P_a \simeq \frac{\gamma}{\gamma-1}\frac{P}{T}\Delta T \Rightarrow P_\ell = P_{iv} + \frac{\gamma}{\gamma-1}\frac{P_{iv}}{T_0}\Delta T$$

for saturation curve: $\Delta P_s = P_\ell - P_{is}$

$$\Delta P_s = \frac{\mu L}{R}\frac{P}{T^2}\cdot\Delta T \Rightarrow P_\ell = P_{is} + \frac{\mu L}{R}\frac{P_{is}}{T_i^2}\Delta T$$

$$\Rightarrow \underbrace{P_{is} - P_{iv} = \left(-\frac{\mu L}{R}\frac{P_{is}}{T_i} + \frac{\gamma}{\gamma-1}P_{iv}\right)\frac{\Delta T}{T_i}}_{\text{linear equation}}$$

$P_{is} = 1{,}94\ \text{kPa}$; $P_{iv} = 1{,}61\ \text{kPa}$, $T_i = 290\ \text{K} \Rightarrow$

$$\Delta T = -3{,}19\,^\circ\text{C} \Rightarrow T_\ell = 13{,}8\,^\circ\text{C}$$