PHYSICS OF INDUCTION COOKING

Solution of the Experimental Problem
Asian Physics Olympiad, Dhahran Saudi Arabia 2025

Oki Gunawan, Herry Kwee, Hendra Kwee, Yendi, M. Morteza Mudrick

v.3.0 2025/5/9

1. EXP#1: CHARACTERIZATION OF THE INDUCTION COIL

1.1 Series RLC circuit

To determine L using resonance experiment we setup a series R-L-C circuit as shown below. We measure the source voltage from the Function Generator (FG): V_S and measure the current I by measuring the voltage across "shunt resistance" R_1=1 Ω: $I = V_{R1} / R_1$.

We use the digital oscilloscope to measure the voltage. For convenient and quick measurement, we can fix one probe terminal (e.g.) on node #1 and repetitively measure voltage on node#2 and node#3 to measure source voltage V_S and V_{R1} repetitively.

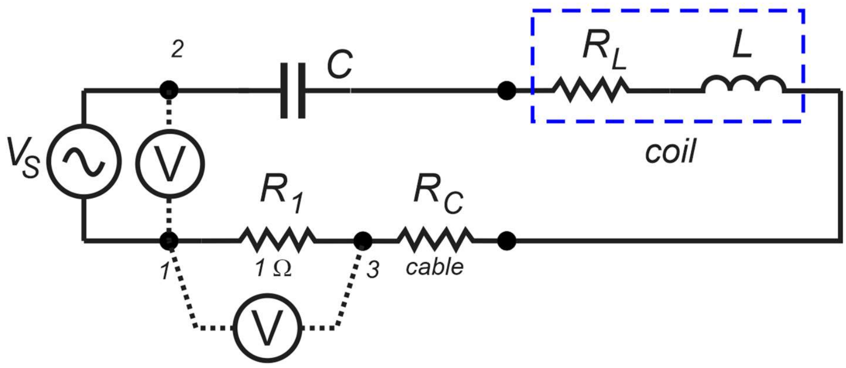


Figure 1. *The series RLC circuit*

The cables involved in the circuit are two pieces of banana-to-pins cable (item#6). Using ohmmeter we obtain: R_C = 0.09 Ω.

1.2 Resonance series RLC circuit

Then we determine the resonant condition where the impedance $Z = V_S / I$ reaches minimum or conductance $G = I / V_S$ reaches maximum. A smart student should quickly scan the frequency first to quickly find the resonant condition and suitable frequency range before taking data, e.g. fix V_S and scan I as a function of frequency. We note that V_S could vary due changing load impedance. The data is shown below:

SOLUTION

	C(Y)	f(X)	VS(Y)	VR1(Y)	G(Y)
lame					
Units	uF	kHz	V	V	S
nents			(Vmax)	(Vmax)	I/VS
1	0.47	16.3	11.987	0.746	0.06223
2		21.6	9.133	0.967	0.10588
3		23.9	4.376	0.616	0.14077
4		28.4	1.465	0.373	0.25461
5		31.6	0.799	0.327	0.40926
6		32.5	0.742	0.32	0.43127
7		33	2.359	1.187	0.50318
8		33.8	2.283	1.157	0.50679
9		35.3	0.685	0.335	0.48905
10		35.6	2.436	1.149	0.47167
11		36.7	1.351	0.544	0.40266
12		37.5	0.894	0.32	0.35794
13		38.8	1.047	0.316	0.30181
14		40.6	1.332	0.335	0.2515
15		43.2	1.922	0.38	0.19771
16		48	3.235	0.464	0.14343
17		54.1	7.915	0.822	0.10385
18		64.8	4.719	0.373	0.07904

	C(Y)	f(X)	VS(Y)	VR1(Y)	G(Y)
me					
nits	uF	Hz	V	V	S
nts			(Vmax)	(Vmax)	I/VS
1	2200	20	4.034	0.936	0.23203
2		31	5.099	1.674	0.3283
3		53	5.023	2.207	0.43938
4		79	4.833	2.474	0.5119
5		109	4.795	2.55	0.5318
6		228	4.643	2.835	0.6106
7		282	4.643	2.892	0.62287
8		305	4.643	2.852	0.61426
9		328	4.643	2.854	0.61469
10		369	4.643	2.854	0.61469
11		463	4.567	2.854	0.62492
12		570	4.567	2.816	0.6166
13		710	4.49	2.816	0.62717
14		877	4.414	2.74	0.62075
15		1070	4.338	2.664	0.61411
16		1390	4.186	2.55	0.60917
17		1730	4.034	2.397	0.5942
18		2170	3.844	2.207	0.57414
19		3030	3.539	1.903	0.53772
20		4360	3.254	1.522	0.46773

Table 1. *RLC resonance data for C=470 nF and C=2200 μF*

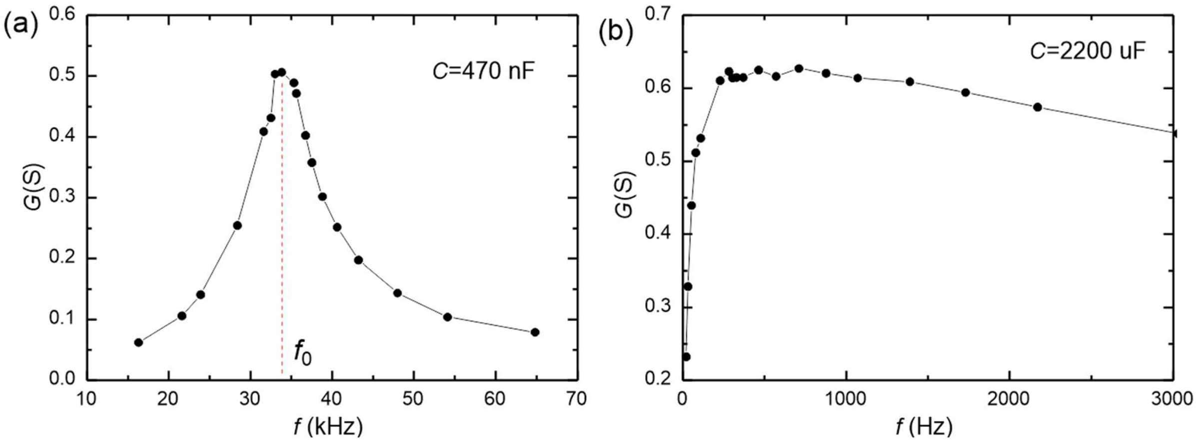


Figure 2. *Resonance conductance plot of the RLC circuit with: (a) C = 470 nF, (b) C = 2200 μF.*

	C(X)	f0(Y)	L(Y)
Units	uF	Hz	uH
mments		resonance	
1	0.47	33800	47.17
2	2200	400	71.96

Table 2. *Results of L determination from RLC resonance*

SOLUTION

The results are shown in Table 2. The resonance frequency is given as: $\omega_0 = 1/\sqrt{LC}$, thus $L = 1/\omega_0^{\,2} C$. We note that the resonance data for C = 470 nF is nice and sharp and yields correct value of L = 47.2 µH, while the data for C = 2200 µF shows broad and poor resonance thus yield inaccurate value of L = 72 µH. This happens because for a series RLC circuit the quality (Q) factor is given as: $Q = \sqrt{L/C}\,/\,R$, therefore smaller capacitance yields a higher quality factor or sharper resonance curve.

1.3 Alternative model to extract *L* and *R*L

We can formulate the impedance as:

$$Z = R_T + j(\omega L - 1/\omega C)\,, \tag{1}$$

$$Z^2 = R_T^{\,2} + (\omega L - 1/\omega C)^2 = R_T^{\,2} + \omega^2 L^2 - 2L/C + 1/\omega^2 C^2\,, \tag{2}$$

$$Z^2 - 1/\omega^2 C^2 = (R_T^{\,2} - 2L/C) + \omega^2 L^2 \tag{3}$$

where the total resistance is: $R_T = R_1 + R_C + R_L$, with R_L is the coil internal resistance. We can linearize the last equation as: $y = a + b\,x$, where: $y = Z^2 - 1/\omega^2 C^2$, $x = \omega^2$, $b = L^2$ and $a = R_T^{\,2} - 2L/C$. Note that since the inductor impedance dominate at high frequency we can also ignore the capacitance term: $1/\omega^2 C^2$ in the analysis.

We can solve for L and R_L as:

$$L = \sqrt{b} \tag{4}$$

$$R_L = \sqrt{a + 2L/C} - R_1 - R_C \tag{5}$$

1.4 RLC experiment to extract *L* and *R*L with C=470 uF and 1000 uF

We perform RLC experiments for C = 470 µF and 1000 µF:

	C(Y)	f(X1)	VS(Y1)	VR1(Y1)	w2(X2)	y(Y2)
ame						
Inits	uF	Hz	V	V	(rad/s)^2	Ohm
ents			(Vmax)	(Vmax)	w^2	Z-1/(w^2C^2
1	470	481	7.516	3.844	9.134E+06	3.3274
2		635	7.326	3.996	1.592E+07	3.07673
3		810	7.23	4.034	2.590E+07	3.03744
4		946	7.04	4.034	3.533E+07	2.91747
5		1090	6.945	3.958	4.690E+07	2.98237
6		1280	6.65	3.844	6.468E+07	2.9228
7		1500	6.469	3.729	8.883E+07	2.9585
8		2140	5.994	3.311	1.808E+08	3.25225
9		2550	5.708	3.082	2.567E+08	3.41243
10		3220	5.233	2.664	4.093E+08	3.84757
11		3810	4.947	2.474	5.731E+08	3.99048

	C(Y)	f(X1)	VS(Y1)	VR1(Y1)	w2(X2)	y(Y2)
Name						
Units	uF	Hz	V	V	(rad/s)^2	Ohm
nents			(Vmax)	(Vmax)	w^2	Z-1/(w^2C^2)
1	1000	362	7.154	4.172	5.173E+06	2.74712
2		463	7.116	4.11	8.463E+06	2.87954
3		538	7.04	4.11	1.143E+07	2.8465
4		649	6.964	4.11	1.663E+07	2.81087
5		761	6.866	4.072	2.286E+07	2.79936
6		839	6.812	4.034	2.779E+07	2.81554
7		959	6.736	3.958	3.631E+07	2.86882
8		1270	6.507	3.805	6.367E+07	2.9088
9		1860	6.013	3.387	1.366E+08	3.14443
10		2550	5.518	2.968	2.567E+08	3.4526
11		3760	4.871	2.359	5.581E+08	4.26185
12		4690	4.567	2.055	8.684E+08	4.93784

***Table** 3. RLC measurements for C = 470 µF and 1000 µF*

SOLUTION

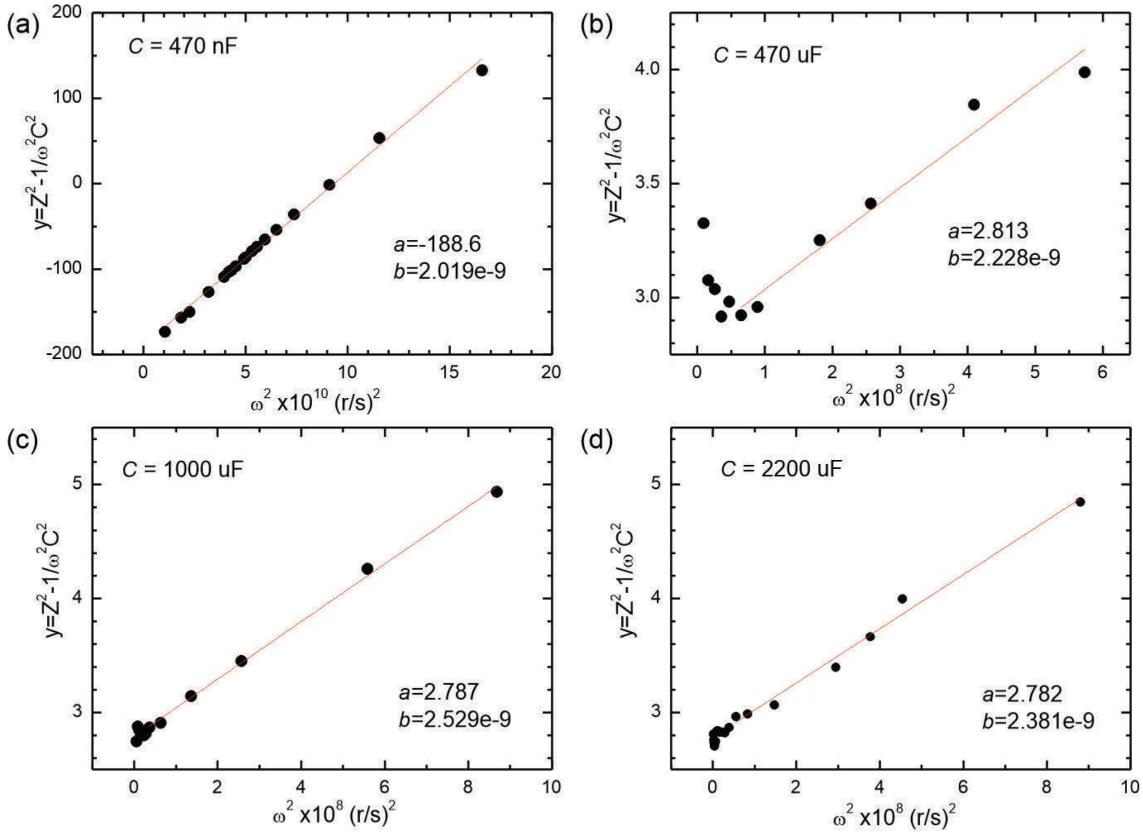


Figure 3. *Extraction of R_L and L with C = 470 nF, 470 μF, 1000 μF and 2200 μF.*

Here are the results:

	C(X)	a(Y)	b(Y)	L(Y)	RL(Y)
Units	uF	Ohm^2	H^2	uH	Ohm
ıents					
1	0.47	-188.6	2.019E-9	44.93	0.524
2	470	2.813	2.228E-9	47.20	0.646
3	1000	2.787	2.529E-9	50.29	0.609
4	2200	2.503	2.737E-9	52.32	0.507

Table 4. *Results of L and R_L determination*

We have average: L = 48.7 μH and average coil resistance: R_L = 0.57 Ω. This is consistent with the original specification of the coil (Wurth Elektronik 760308101303): L = 47 μH and R_L = 0.46 Ω.

Therefore this second technique is more accurate in determining L even in the case where the resonance is poor. We obtain R_L as a “bonus” from the analysis as it comes from the linear fit “intercept” however they are less accurate.

SOLUTION

Measurement of R_L directly with the multimeter for verification is also acceptable, we obtain: $R_L = (0.47 \pm 0.03)\ \Omega$.

2. EXP#2: MUTUAL INDUCTION AND SKIN DEPTH

2.A Mutual Induction

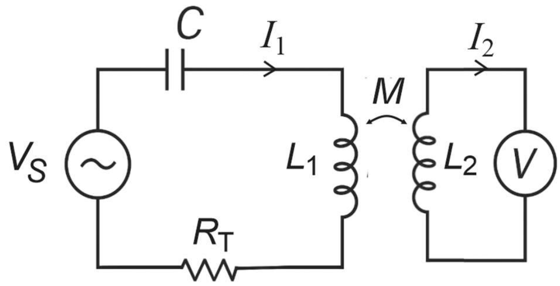


Figure 4. *Mutual inductance setup*

2.2 Mutual inductance experiment

Here we perform the measurement twice where the primary is coil#1 and secondary is coil#2 and then we swap them.

$$V_2 = -M\frac{di_1}{dt} + L_2\frac{di_2}{dt} = -M\frac{di_1}{dt} \tag{6}$$

$$Z' = \frac{V_2}{I_1} = -j\omega M \tag{7}$$

The self inductance contribution is negligible since the second coil is connected to voltmeter and $i_2 \sim 0$. Then we can tabulate the "impedance" $Z = V_2 / I_1$ as a function of frequency and extract M.

	f(X)	I1(Y)	V2(Y)	Z(Y)
Units	Hz	A	V	Ohm
ments				
1	1020	3.539	0.096	0.02713
2	1990	3.082	0.184	0.0597
3	3000	2.588	0.241	0.09312
4	4040	2.131	0.278	0.13046
5	5010	1.827	0.295	0.16147
6	6000	1.617	0.312	0.19295
7	7010	1.427	0.32	0.22425
8	8060	1.256	0.342	0.27229
9	9140	1.104	0.35	0.31703
10	10100	1.012	0.358	0.35375

	f(X)	I2(Y)	V1(Y)	Z(Y)
Inits	Hz	A	V	Ohm
ents				
1	1010	3.52	0.136	0.03864
2	2000	3.006	0.2	0.06653
3	3000	2.512	0.27	0.10748
4	4000	2.112	0.299	0.14157
5	5000	1.808	0.316	0.17478
6	6060	1.56	0.337	0.21603
7	7020	1.355	0.339	0.25018
8	8060	1.21	0.358	0.29587
9	9010	1.104	0.369	0.33424
10	10200	0.997	0.373	0.37412

Table 5. *Mutual inductance determination*

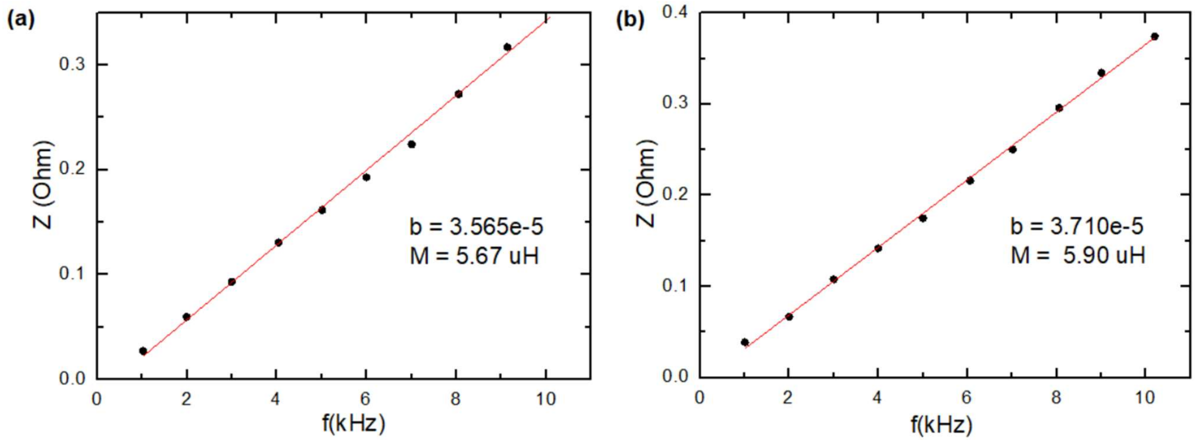


Figure 5. *Mutual inductance determination results: (a) Primary=Coil#1, Secondary=Coil#2 (b) Primary=Coil#2, Secondary=Coil#1.*

2.3 Mutual inductance results

We fit the data to linear equation: $y = a + b\,x$. Here we obtain the mutual inductance: $M = b / 2\pi$, and we obtain M reasonably close results between the two measurements: $M_{1\text{-}2}$ = 5.67 µH and $M_{2\text{-}1}$ = 5.90 µH with average: M = 5.79 µH.

2.B Skin depth experiment

2.4 Skin depth equations model and experiments

<u>Equation model to determine *n*:</u>

We can perform two-step linear regressions to extract n and σ as follows. For each metal we perform measurement of the voltage in secondary coil (coil#1) which is proportional to B after it passes through the metals:

$$V_2 \sim B(z) = B_0 \exp(-z / \delta) = B_0 \exp(-N\, t_0 / \delta(f)) \tag{8}$$

where t_0 is the metal thickness and N is the number of metal. Therefore we expect the voltage in the secondary voltage will drop more metal plates. Therefore we can extract the skin depth at frequency f from:

$$\ln V_2 = -t_0 / \delta(f) \times N + c_0 \tag{9}$$

where c_0 is a constant that we ignore. We can determine the skin depth at a frequency f using :

$$\delta(f) = -t_0 / b_1 \tag{10}$$

where b_1 is the slope of ln(V_2) vs. N data.

Then we repeated this analyis at different frequencies, using Eq. 2 of the problem set:

$$\ln \delta = \frac{n}{2} \ln f + \frac{\ln(\sigma^m / \pi\mu)}{2} \tag{11}$$

Using linear model: $y = a_2 + b_2 x$ of ln δ vs. ln f, we can obtain:

$$n = 2 \frac{\Delta \ln \delta}{\Delta \ln f} = 2b_2 \tag{12}$$

The conductivity (using m = -1 as determined in Q2.5 later) is given as:

$$\sigma = \frac{\exp(-2a_2)}{\pi\mu} \tag{13}$$

Note: See Appendix A at the end for an alternative single regression analysis which is also valid but less accurate.

Skin Depth Experiments:

We inject the oscillating current to coil#1 and measure the induced voltage at the coil#2 while keep adding the metal pieces. The voltage in coil#2 is proportional to the magnetic field generated from coil#1 after attenuated by the metal pieces.

The student is expected to test first range of appropriate frequencies before taking data. To obtain the best results the student should perform the experiment with all 5 or 4 plates for each metal and repeat that at minimum 5 frequencies. The results are shown below.

(1) Aluminum:

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V		
nents			# plate	
1	1010		0	--
2	1010	0.103	1	-2.273
3	1010	0.068	2	-2.688
4	1010	0.053	3	-2.937
5	1010	0.046	4	-3.079
6	1010	0.037	5	-3.297
7	1520		0	--
8	1520	0.114	1	-2.172
9	1520	0.084	2	-2.477
10	1520	0.064	3	-2.749
11	1520	0.05	4	-2.996
12	1520	0.036	5	-3.324
13	2010		0	--
14	2010	0.129	1	-2.048
15	2010	0.081	2	-2.513
16	2010	0.057	3	-2.865
17	2010	0.046	4	-3.079
18	2010	0.036	5	-3.324
19	2660		0	--
20	2660	0.129	1	-2.048
21	2660	0.081	2	-2.513
22	2660	0.054	3	-2.919
23	2660	0.043	4	-3.147
24	2660	0.031	5	-3.474

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V		
ments			# plate	
25	3140		0	--
26	3140	0.16	1	-1.833
27	3140	0.093	2	-2.375
28	3140	0.063	3	-2.765
29	3140	0.046	4	-3.079
30	3140	0.034	5	-3.381
31	3540	0.342	0	-1.073
32	3540	0.16	1	-1.833
33	3540	0.107	2	-2.235
34	3540	0.08	3	-2.526
35	3540	0.049	4	-3.016
36	3540	0.034	5	-3.381
37	4000		0	--
38	4000	0.169	1	-1.778
39	4000	0.091	2	-2.397
40	4000	0.063	3	-2.765
41	4000	0.04	4	-3.219
42	4000	0.02	5	-3.912
43	4520		0	--
44	4520	0.335	1	-1.094
45	4520	0.167	2	-1.790
46	4520	0.099	3	-2.313
47	4520	0.065	4	-2.733
48	4520	0.034	5	-3.381

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	lndelta(X3)
Units	Hz	-	mm	mm	ln(Hz)	ln(m)
nents		slope			ln(f)	ln(delta)
1	1010	-0.2439	2.99303	0.73	6.918	-5.811
2	1520	-0.2824	2.58499		7.326	-5.958
3	2010	-0.3118	2.34094		7.606	-6.057
4	2660	-0.3485	2.09475		7.886	-6.168
5	3140	-0.3802	1.92026		8.052	-6.255
6	3540	-0.4395	1.66091		8.172	-6.400
7	4000	-0.5090	1.4341		8.294	-6.547
8	4520	-0.5519	1.32267		8.416	-6.628

SOLUTION

(a) (b)

Table 6. *Skin depth experiment for Al: (a) Raw data, (b) Power factor n and σ analysis.*

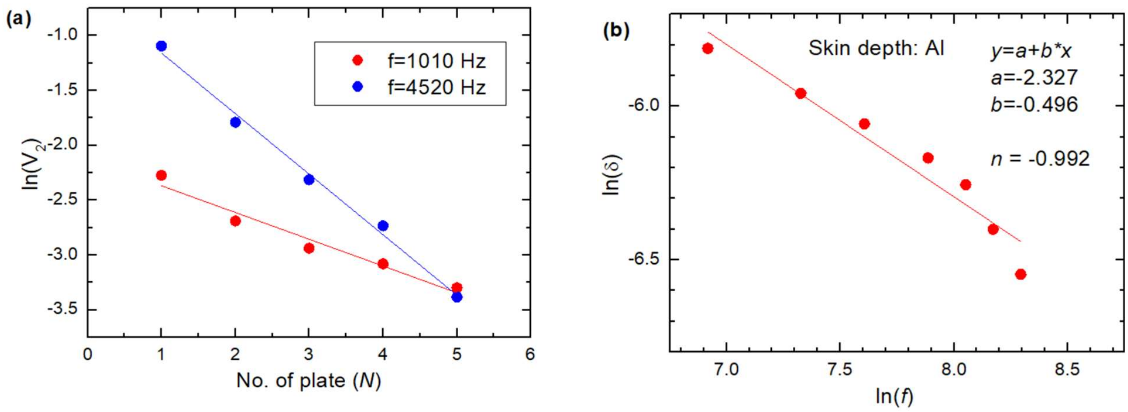


Figure 6. *Skin depth analysis for Al: (a) Voltage at secondary coil vs. number of plate N at two extreme frequencies (b) Skin depth vs. frequency to determine power factor n and σ.*

(2) Copper:

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V		
Comments			# plate	
1	990		0	--
2	990	0.074	1	-2.604
3	990	0.043	2	-3.147
4	990	0.034	3	-3.381
5	990	0.026	4	-3.650
6	990	0.02	5	-3.912
7	1520		0	--
8	1520	0.074	1	-2.604
9	1520	0.043	2	-3.147
10	1520	0.031	3	-3.474
11	1520	0.02	4	-3.912
12	1520	0.014	5	-4.269
13	2030		0	--
14	2030	0.083	1	-2.489
15	2030	0.04	2	-3.219
16	2030	0.023	3	-3.772
17	2030	0.02	4	-3.912
18	2030	0.011	5	-4.510
19	2510		0	--
20	2510	0.083	1	-2.489
21	2510	0.043	2	-3.147
22	2510	0.029	3	-3.540
23	2510	0.017	4	-4.075
24	2510	0.011	5	-4.510
25	3000		0	--
26	3000	0.091	1	-2.397
27	3000	0.041	2	-3.194
28	3000	0.022	3	-3.817
29	3000	0.014	4	-4.269
30	3000	0.01	5	-4.605
31	4000		0	--
32	4000	0.099	1	-2.313
33	4000	0.046	2	-3.079
34	4000	0.026	3	-3.650
35	4000	0.012	4	-4.423
36	4000	0.008	5	-4.828

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	lndelta(X3)
Units	Hz	-	mm	mm	ln(Hz)	ln(m)
Comments		slope			ln(f)	ln(delta)
1	990	-0.3120	2.244	0.7	6.898	-6.100
2	1520	-0.4095	1.709		7.326	-6.372
3	2030	-0.4735	1.478		7.616	-6.517
4	2510	-0.4970	1.408		7.828	-6.565
5	3000	-0.5491	1.275		8.006	-6.665
6	4000	-0.6375	1.098		8.294	-6.814

(a) (b)

Table 7. *Skin depth experiment for Cu: (a) Raw data, (b) Power factor n and σ analysis*

SOLUTION

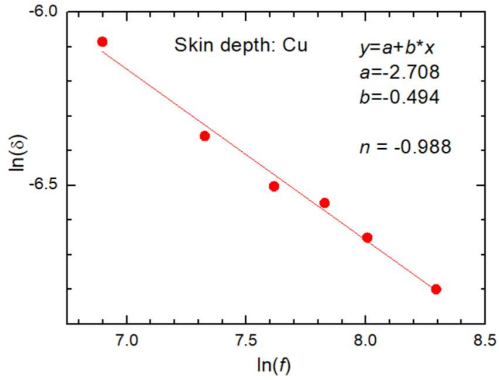


Figure 7. *Skin depth analysis for Cu*

(3) SS304:

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V		
ments			# plate	
1	15000		0	--
2	15000	0.61	1	-0.4943
3	15000	0.46	2	-0.7765
4	15000	0.39	3	-0.9416
5	15000	0.34	4	-1.079
6	15000		5	--
7	20000		0	--
8	20000	0.647	1	-0.4354
9	20000	0.571	2	-0.5604
10	20000	0.411	3	-0.8892
11	20000	0.342	4	-1.073
12	20000		5	--
13	25000		0	--
14	25000	0.723	1	-0.3243
15	25000	0.51	2	-0.6733
16	25000	0.392	3	-0.9365
17	25000	0.327	4	-1.118
18	25000		5	--
19	30100		0	--
20	30100	0.761	1	-0.2731
21	30100	0.518	2	-0.6578
22	30100	0.403	3	-0.9088
23	30100	0.331	4	-1.106
24	30100		5	--

	f(X)	V2(Y)	N(Y)	lnV2(Y)
Units	Hz	V		
ments			# plate	
25	35000		0	--
26	35000	0.79	1	-0.2357
27	35000	0.53	2	-0.6349
28	35000	0.4	3	-0.9163
29	35000	0.31	4	-1.171
30	35000		5	--
31	40000		0	--
32	40000	1.134	1	0.1258
33	40000	0.715	2	-0.3355
34	40000	0.525	3	-0.6444
35	40000	0.411	4	-0.8892
36	40000		5	--
37	45800		0	--
38	45800	1.18	1	0.1655
39	45800	0.731	2	-0.3133
40	45800	0.54	3	-0.6162
41	45800	0.42	4	-0.8675
42	45800		5	--
43	51300		0	--
44	51200	0.525	1	-0.6444
45	51200	0.297	2	-1.214
46	51200	0.217	3	-1.528
47	51200	0.194	4	-1.640
48	51200		5	--

(a)

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	lndelta(X3)
Units	Hz		mm	mm	ln(Hz)	ln(m)
ents		slope			ln(f)	ln(delta)
1	15000	-0.1919	3.752	0.72	9.616	-5.586
2	20000	-0.2242	3.212		9.903	-5.741
3	25000	-0.2644	2.723		10.13	-5.906
4	30100	-0.2750	2.618		10.31	-5.945
5	35000	-0.3087	2.332		10.46	-6.061
6	40000	-0.3354	2.147		10.60	-6.144
7	45800	-0.3402	2.116		10.73	-6.158
8	51200	-0.3301	2.181		10.84	-6.128

(b)

Table 8. *Skin depth experiment for SS304: (a) Raw data, (b) Power factor n and σ analysis.*

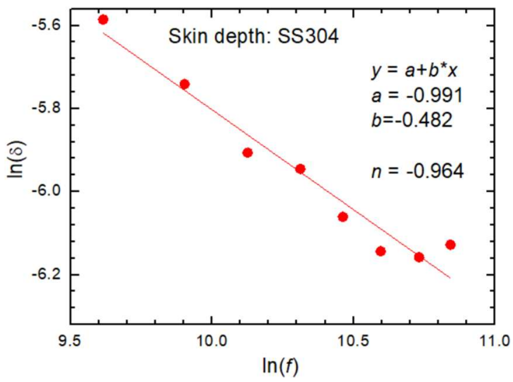


Figure 8. *Skin depth analysis for SS304: Skin depth vs. frequency to determine power factor n and σ.*

(4) SS410:

	f(X1)	V2(Y1)	N(X2)	lnV2(Y2)
Jnits	Hz	V		
ents			# plate	
1	843	2.05	0	--
2	843	0.68	1	-0.3857
3	843	0.61	2	-0.4943
4	843	0.53	3	-0.6349
5				--
6	1070		0	--
7	1070	0.02	1	-3.912
8	1070	0.02	2	-3.912
9	1070	0.02	3	-3.912
10				--
11	2210	3.81	0	--
12	2210	0.99	1	-0.01005
13	2210	0.84	2	-0.1744
14	2210	0.76	3	-0.2744
15				--
16	43200		0	--
17	43200	0.753	1	-0.2837
18	43200	0.571	2	-0.5604
19	43200	0.441	3	-0.8187
20				--
21	60000	2.19	0	--
22	60000	0.36	1	-1.022
23	60000	0.2	2	-1.609
24	60000	0.14	3	-1.966
25	60000	0.1	4	-2.303
26	60000	0.07	5	-2.659

(a)

	f(X1)	b(Y1)	delta(Y1)	t(Y1)	lnf(X2)	lndelta(X3)
Units	Hz		mm	mm		
nents		slope			ln(f)	ln(delta)
1	843	-0.1246	6.100	0.76	6.737	1.808
2	1070	--	--		6.975	--
3	2210	-0.1322	5.750		7.701	1.749
4	43200	-0.2675	2.841		10.674	1.044
5	60000	-0.3968	1.915		11.002	0.650

(b)

Table 9. *Skin depth experiment for SS410: (a) Raw data, (b) Power factor n and σ analysis.*

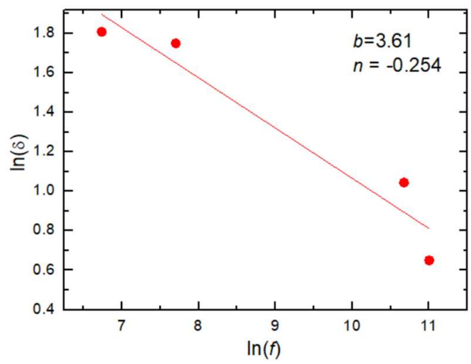


Figure 9. *Skin depth analysis for SS410*

We note that SS410 behaves differently, the output signal V_2 drops greatly upon insertion of the metals. Upon final analysis it yields n = -0.5 which is not correct.

Apparently because SS410 is magnetic, its skin depth is too small and almost no magnetic field could penetrate the metal and the fringing field around the metal becomes dominant thus the “skin depth” measurement becomes anomalous. Thus **SS410 is the metal with "extreme skin depth"** and is excluded in the subsequent analysis.

The summary of the skin depth experiment is shown below:

	Metal(X)	a(Y)	b(Y)	n(Y)	sigma(Y)	sigmaR(Y)	sigmaEr
ame							
Jnits					S/m	S/m	%
ents		intercept	slope			Ref lit.	Error
1	Al	-2.327	-0.496	-0.992	2.66E+07	3.7E7	-28.1
2	Cu	-2.708	-0.494	-0.988	5.70E+07	5.88E7	-3.1
3	SS304	-0.991	-0.481	-0.962	1.84E+06	1.39E6	32.3

Table 10. *Summary results of the skin depth experiment. "sigmaErr" is the percentage error of the measured conductivity vs. the reference literature values ("sigmaR")*

We observe from all three metals we obtain consistent frequency power factor with average n = -0.98, and thus $\boldsymbol{n}$ **= -1** (rounded to nearest integer).

2.5 **Conductivity power factor *m***

Since we have obtained n = -1, from the Eq. 2 in the problem set:

$$\delta^2 = \frac{\sigma^m}{\pi\mu f} \quad \text{or} \quad [\sigma]^m = [\delta]^2[\mu][f] \tag{14}$$

Using dimensional or unit analysis: [σ] = 1/Ω.m = A/V.m, [δ] = m, [μ] = H/m=V.s/A.m and [f] =1/s, we have:

$$[A/V.m]^{m} = [m]^2[V.s/A.m][1/s] = [V.m/A] \tag{15}$$

Thus we get $\boldsymbol{m}$ = -1, and the final skin depth formula is:

$$\delta = \frac{1}{\sqrt{\pi\sigma\mu f}} \tag{16}$$

2.6 **Conductivity of the metals**

The conductivity of the metals can be calculated using Eq. (13) and the results are shown in Table 10. We observe that our measured conductivity is reasonably good to the reference literature values (within +/- 30% error). The larger uncertainty is due to the results originating from a value that depends exponentially on the intercepts [Eq. (13)].

Note: This method provides a very attractive approach to perform conductivity measurement in a material because it is non-contact.

SOLUTION

3. EXP #3: COOKING, SPECIFIC HEAT CAPACITY AND EFFECTIVE LOAD RESISTANCE

3.1 **The induction cooking operating principle**

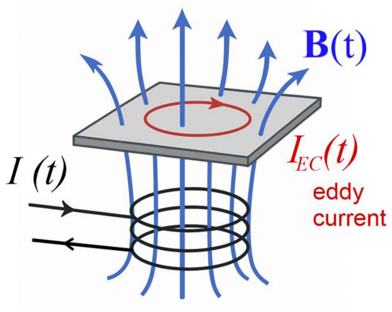


Figure 10. *Principle of induction cooker*

Operating principle:
Oscillating current drive to the coil → generate oscillating magnetic field → generate eddy current in the plate → generate Joule heating in the plate

3.2 **Specific Heat of the metal pan**

We developed a model that allow us to extract the specific heat capacity of the metal pan. The heat transfer energy balance can be modeled as total power input to the cooking pan is equal to the heating rate of the pan and radiation. We ignore convection losses as indicated in the problem.

$$P_{IN} = m\,c\,dT/dt + e\,A\,\sigma_S (T^4 - T_0^{\,4}) \tag{17}$$

where m is the mass of the metal pan, c is the specific heat, T is the plate temperature, T_0 is the ambient temperature, e is the emissivity, A is the surface area of the radiating body and σ_S is the Stefan Boltzmann constant.

We need to warm up the cooker first and then turn off the power input to let it cool. The cooling behavior is given as:

$$T^4 = -\frac{m\,c}{e\,A\,\sigma_S}\frac{dT}{dt} + T_0^{\,4} = -\frac{\rho\,c\,t_0}{2e\,\sigma_S}\frac{dT}{dt} + T_0^{\,4} \tag{18}$$

We note that the factor of two comes from consideration that that the radiation area is twice the surface area of the metal i.e. $A = 2WL$, where W and L is the width and the length of the "pan", t_0 is the metal thickness. We perform linear regression: $y = a + b\,x$, with x is dT/dt and y is T^4 and we can ignore the effect of T_0.

The specific heat can be calculated as:

$$c = -\frac{2\,e\,\sigma_S\,b}{\rho\,t_0} \tag{19}$$

Note: It is possible to solve the differential equation in the Eq. (18), but the solution requires the knowledge of starting temperature T_0 which could vary with repeated experiments, thus such solution is not practical.

3.3 **Specific heat of the Aluminum pan**

We then measure the thermistor resistor (R_{NTC}) and calculate the pan temperature T using Eq. (4) in the problem set. Specifically, we need to derive:

$$T = \left[\frac{\ln(R/R_0)}{B} + \frac{1}{T_0}\right]^{-1} \tag{20}$$

We record ambient temperature is $T = 306.5$ K = 33.35 C, for completeness but this does not impact subsequent analysis. We can calculate the derivate dT/dt at point n numerically as:

$$\frac{dT_n}{dt} = \frac{T_{n+1} - T_{n-1}}{t_{n+1} - t_{n-1}} \tag{21}$$

We heat up the "pan" approximately for 1 min until the temperature reaches 325.4 K (52.3 °C) which marks $t = 0$ s and record the NTC resistance as a function of time as the "pan" cools.

SOLUTION

	te(X1)	RNTC(Y1)	T(Y1)	dTdt(X2)	T4(Y2)
Units	s	kOhm	K	K	K^4
nments	time				T^4
1	--	6.97	306.5	--	--
2	0	3.302	325.4	--	1.121E+10
3	20	3.532	323.6	-0.085	1.096E+10
4	40	3.753	322.0	-0.075	1.075E+10
5	60	3.961	320.6	-0.0625	1.056E+10
6	80	4.123	319.5	-0.06	1.042E+10
7	100	4.334	318.2	-0.0575	1.026E+10
8	120	4.513	317.2	-0.0475	1.012E+10
9	140	4.667	316.3	-0.0425	1.002E+10
10	160	4.818	315.5	-0.0375	9.914E+09
11	180	4.957	314.8	-0.0325	9.824E+09
12	200	5.086	314.2	-0.03	9.744E+09
13	220	5.204	313.6	-0.0275	9.673E+09
14	240	5.318	313.1	-0.025	9.607E+09
15	260	5.426	312.6	-0.0225	9.546E+09
16	280	5.519	312.2	-0.0225	9.495E+09
17	300	5.611	311.7	-0.02	9.445E+09
18	320	5.694	311.4	-0.015	9.402E+09
19	340	5.771	311.1	-0.0175	9.362E+09
20	360	5.844	310.7		9.325E+09

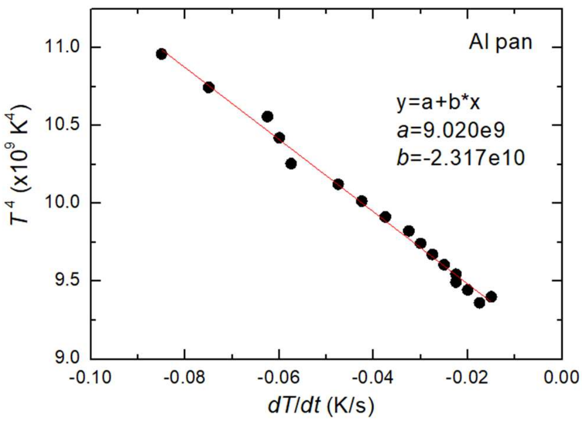


Figure 11. *Specific heat measurement for Al pan*

Using Eq. (19), $e = 0.65$, $\rho = 2700$ kg/m^3, $t_0 = 0.71$ mm, we obtain slope $b = -2.317\times10^{10}$ and specific heat $c = 890$ J/kg.K. The literature value is $c_{Al} = 900$ J/kg.K.

Note: In this Olympiad problem, the emissivity e value is chosen yield c close to the reference value.

3.4 **Specific heat of the SS410 pan**

	tc(X1)	RNTC(Y1)	T(Y1)	dTdt(X2)	T4(Y2)
Units	s	kOhm	K	K	K^4
ments					T^4
1		6.97	306.50	--	8.825E+09
2	0	4	320.30	--	1.053E+10
3	20	4.02	320.17	-0.02000	1.051E+10
4	40	4.127	319.49	-0.03750	1.042E+10
5	60	4.26	318.68	-0.04250	1.031E+10
6	80	4.409	317.79	-0.04500	1.020E+10
7	100	4.565	316.91	-0.04000	1.009E+10
8	120	4.703	316.15	-0.03750	9.990E+09
9	140	4.851	315.37	-0.03750	9.892E+09
10	160	4.975	314.74	-0.03250	9.813E+09
11	180	5.095	314.14	-0.02750	9.738E+09
12	200	5.205	313.61	-0.02500	9.673E+09
13	220	5.312	313.10	-0.02250	9.610E+09
14	240	5.406	312.67	-0.02000	9.557E+09
15	260	5.496	312.26	-0.02000	9.507E+09
16	280	5.58	311.88	-0.01750	9.462E+09
17	300	5.654	311.56	-0.01500	9.422E+09
18	320	5.719	311.28	-0.01500	9.389E+09
19	340	5.788	310.99	-0.01250	9.353E+09
20	360	5.843	310.75		9.325E+09

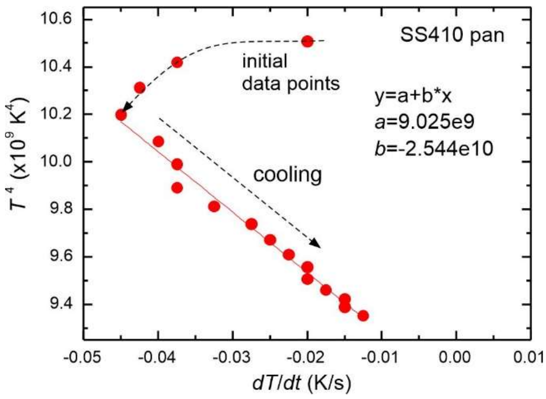


Figure 12. *Specific heat measurement for SS410 pan*

SOLUTION

We note sometimes, like for SS410 here, the initial data do not form a straight line as the system has not reached a steady state, thus we only perform the analysis on the linear segment as expected from the model.

Using Eq. (19), $e = 0.8$, $\rho = 7700$ kg/m^3, $t_0 = 0.75$ mm, we obtain slope $b = -2.544\times10^{10}$ and specific heat $c = 400$ J/kg.K. The literature value is $c_{SS410} = 460$ J/kg.K.

3.5 R_{LOAD} for Aluminum pan

Now we model that the pan appears as "load resistance" to the primary circuit. The power input given to the metal pan will increase the temperature of the "pan" and also radiate to the surrounding:

$$P_{IN} = I^2 R_{LOAD} = m\,c\,dT/dt + e\,A\,\sigma_S (T^4 - T_0^{\;4}) \tag{22}$$

For later analysis, we can rearrange this to:

$$m\,c\,dT/dt + eA\,\sigma_S T^4 = I^2 R_{LOAD} + eA\,\sigma_S T_0^{\;4} \tag{23}$$

$$P'_{TOT} = P_C + P_{RAD}' = I^2 R_{LOAD} + P_{RAD,0} \tag{24}$$

where: $P_C = m\,c\,dT/dt$, $P'_{RAD} = eA\sigma_S T^4$ and $P_{RAD,0} = eA\sigma_S T^4$

We can perform linear fit of the experimental data of P_{TOT}' vs. I^2 following linear equation: $y = a + b\,x$, where: $y = P_{TOT}'$, $x = I^2$, $b = R_{LOAD}$ and $a = P_{RAD,0}$, which we assume to be constant and can be ignored.

Thus, we can obtain R_{LOAD} from the linear fit of P_{TOT}' vs. I^2. Note: the current I must be of RMS value since it is an AC current.

<u>Aluminum pan</u>:

We now perform the "cooking" experiment on the Al "pan". We will vary the current to the circuit and monitor the heating behavior.

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Jnits	A	A	Ohm	K	K/s	W	W	W	W
ents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1	--	Ambient	--	--	--	--	--	--	--
2	0.4172	0	6.6	307.80	--	--	0.26466	--	--
3	(0.59 pk)	20	6.546	308.00	8.704E-03	6.098E-03	0.26534	0.27143	0.27158
4	--	40	6.505	308.15	7.483E-03	5.243E-03	0.26586	0.27110	--
5	--	60	6.465	308.30	7.819E-03	5.478E-03	0.26637	0.27185	
6	--	80	6.421	308.46	6.366E-03	4.460E-03	0.26694	0.27140	
7	--	100	6.397	308.55	5.184E-03	3.632E-03	0.26725	0.27088	
8	--	120	6.366	308.67	5.587E-03	3.915E-03	0.26766	0.27157	
9	--	140	6.338	308.78	5.045E-03	3.535E-03	0.26802	0.27156	
10	--	160	6.313	308.87	4.782E-03	3.351E-03	0.26836	0.27171	
11	--	180	6.288	308.97	4.611E-03	3.231E-03	0.26869	0.27192	
12	--	200	6.265	309.06	4.245E-03	2.974E-03	0.269	0.27197	
13	--	220	6.244	309.14	3.874E-03	2.714E-03	0.26928	0.27199	
14	--	240	6.225	309.21	--	--	0.26954	--	
15	--	--	--	--	--	--	--	--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
s	A	A	Ohm	K	K/s	W	W	W	W
s					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.4978	0	6.6	307.80	--	--	0.26466	--	0.27583
3	(0.704 Apk)	20	6.515	308.12	1.437E-02	1.007E-02	0.26573	0.27580	--
4		40	6.444	308.38	1.251E-02	8.763E-03	0.26664	0.27540	
5		60	6.381	308.62	1.114E-02	7.805E-03	0.26746	0.27526	
6		80	6.326	308.82	1.040E-02	7.287E-03	0.26818	0.27547	
7		100	6.272	309.03	9.436E-03	6.611E-03	0.2689	0.27551	
8		120	6.228	309.20	8.550E-03	5.990E-03	0.2695	0.27549	
9		140	6.184	309.37	8.127E-03	5.694E-03	0.2701	0.27579	
0		160	6.145	309.53	7.697E-03	5.393E-03	0.27063	0.27602	
1		180	6.106	309.68	7.353E-03	5.152E-03	0.27117	0.27632	
2		200	6.071	309.82	6.804E-03	4.767E-03	0.27166	0.27643	
3		220	6.038	309.95	6.444E-03	4.515E-03	0.27213	0.27664	
4		240	6.007	310.08	--	--	0.27257	--	
5					--	--		--	

SOLUTION

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
nents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
2	0.5650	0	6.6	307.80	--	--	0.26466	--	0.27863
3	(0.799 Apk)	20	6.503	308.16	1.671E-02	1.171E-02	0.26588	0.27759	--
4		40	6.419	308.47	1.519E-02	1.064E-02	0.26696	0.27761	
5		60	6.341	308.77	1.398E-02	9.793E-03	0.26798	0.27778	
6		80	6.272	309.03	1.301E-02	9.113E-03	0.2689	0.27802	
7		100	6.206	309.29	1.209E-02	8.469E-03	0.2698	0.27827	
8		120	6.148	309.51	1.134E-02	7.946E-03	0.27059	0.27854	
9		140	6.091	309.74	1.076E-02	7.539E-03	0.27138	0.27892	
10		160	6.04	309.95	1.006E-02	7.052E-03	0.2721	0.27915	
11		180	5.991	310.14	9.549E-03	6.690E-03	0.2728	0.27949	
12		200	5.946	310.33	8.916E-03	6.247E-03	0.27344	0.27969	
13		220	5.904	310.50	8.369E-03	5.864E-03	0.27405	0.27992	
14		240	5.865	310.66	--	--	0.27462	--	
15					--	--		--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
nments					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.6590	0	6.6	307.80	--	--	0.26466	--	
3	(0.932 Apk)	20	6.444	308.38	2.608E-02	1.827E-02	0.26664	0.28491	0.28476
4		40	6.32	308.85	2.213E-02	1.551E-02	0.26826	0.28377	--
5		60	6.212	309.26	2.036E-02	1.426E-02	0.26971	0.28398	
6		80	6.111	309.66	1.845E-02	1.292E-02	0.2711	0.28403	
7		100	6.026	310.00	1.655E-02	1.159E-02	0.2723	0.28389	
8		120	5.947	310.32	1.578E-02	1.105E-02	0.27343	0.28448	
9		140	5.872	310.63	1.486E-02	1.041E-02	0.27452	0.28493	
10		160	5.804	310.92	1.370E-02	9.596E-03	0.27553	0.28513	
11		180	5.742	311.18	1.270E-02	8.895E-03	0.27646	0.28536	
12		200	5.685	311.43	1.198E-02	8.395E-03	0.27733	0.28573	
13		220	5.631	311.66	1.135E-02	7.952E-03	0.27817	0.28612	
14		240	5.581	311.88	--	--	0.27895	--	

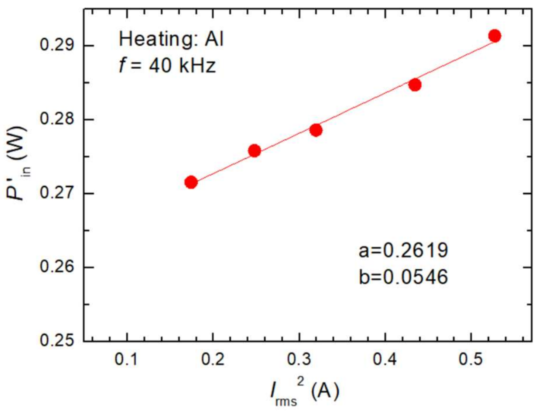
	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
nments					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.7260	0	6.6	307.80	--	--	0.26466	--	
3	(1.027 Apk)	20	6.373	308.65	3.709E-02	2.598E-02	0.26756	0.29355	0.29137
4		40	6.206	309.29	2.998E-02	2.101E-02	0.2698	0.29080	--
5		60	6.065	309.84	2.620E-02	1.836E-02	0.27175	0.29011	
6		80	5.944	310.34	2.388E-02	1.673E-02	0.27347	0.29020	
7		100	5.832	310.80	2.220E-02	1.555E-02	0.27511	0.29067	
8		120	5.732	311.22	2.008E-02	1.407E-02	0.27662	0.29069	
9		140	5.644	311.60	1.839E-02	1.288E-02	0.27797	0.29085	
10		160	5.563	311.96	1.749E-02	1.225E-02	0.27924	0.29149	
11		180	5.486	312.30	1.642E-02	1.150E-02	0.28047	0.29197	
12		200	5.417	312.62	1.495E-02	1.047E-02	0.2816	0.29207	
13		220	5.355	312.90	1.435E-02	1.005E-02	0.28263	0.29268	
14		240	5.293	313.19	--	--	0.28367	--	
15					--	--		--	

Table 11. *Data for R_{LOAD} determination of the Aluminum pan using various current.*

For calculation convenience, we tabulate all the properties of the Al pan as follows:

Quantity	**Symbol**	**Values**
Emissivity	e	0.65
Mass density	ρ	2700 kg/m^3
Heat capacity	c	913.7 J/kg.K
Heat capacity reference	c_{REF}	900 J/kg.K
Radiation area	A	2x2cmx2cm = 8x10^{-4} m^2
Volume	V	2cmx2cmx0.71mm=2.84x10^{-7} m^3
Mass	m	ρV=7.668x10^{-4} kg
Ambient temperature	T_0	303.66 K (R_{NTC0} = 7.864 kΩ)

Table 12. *Properties of the Al pan*



SOLUTION

Figure 13. *"Effective load resistance" measurement of the aluminum pan.*

We then plot $P_{tot'ave}$ vs. I_{rms}^2 as shown above. The slope directly yields the load resistance R_{LOAD} = 54.6 mΩ.

3.6 R_{LOAD} for the SS410 pan

Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
A	A	Ohm	K	K/s	W	W	W	W
				dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
--	--	--	--	--	--	--	--	--
0.4441	0	6	310.11	--	--	0.33559	--	0.35215
(0.628 pk)	20	5.9	310.52	1.816E-02	1.805E-02	0.33737	0.35541	--
--	40	5.824	310.83	1.458E-02	1.448E-02	0.33875	0.35323	--
--	60	5.761	311.10	1.297E-02	1.288E-02	0.33991	0.35279	
--	80	5.702	311.35	1.128E-02	1.121E-02	0.34101	0.35222	
--	100	5.656	311.55	9.555E-03	9.493E-03	0.34189	0.35138	
--	120	5.614	311.73	8.872E-03	8.815E-03	0.34269	0.35151	
--	140	5.575	311.91	8.058E-03	8.007E-03	0.34345	0.35145	
--	160	5.541	312.06	7.116E-03	7.070E-03	0.34411	0.35118	
--	180	5.511	312.19	6.603E-03	6.561E-03	0.3447	0.35126	
--	200	5.482	312.32	6.419E-03	6.378E-03	0.34528	0.35165	
--	220	5.454	312.45	5.775E-03	5.738E-03	0.34584	0.35157	
--	240	5.431	312.55	--	--	0.3463	--	
--	--	--	--	--	--	--	--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
nents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.4978	0	6	310.11	--	--	0.33559	--	0.36340
3	(0.704 pk)	20	5.827	310.82	3.113E-02	3.093E-02	0.33869	0.36962	--
4		40	5.702	311.35	2.481E-02	2.465E-02	0.34101	0.36567	
5		60	5.596	311.81	2.096E-02	2.083E-02	0.34304	0.36386	
6		80	5.511	312.19	1.812E-02	1.800E-02	0.3447	0.36270	
7		100	5.434	312.54	1.637E-02	1.626E-02	0.34624	0.36250	
8		120	5.367	312.85	1.465E-02	1.455E-02	0.3476	0.36216	
9		140	5.307	313.12	1.332E-02	1.324E-02	0.34884	0.36208	
10		160	5.253	313.38	1.194E-02	1.187E-02	0.34998	0.36184	
11		180	5.206	313.60	1.112E-02	1.105E-02	0.35098	0.36202	
12		200	5.16	313.82	1.050E-02	1.044E-02	0.35197	0.36240	
13		220	5.119	314.02	9.752E-03	9.689E-03	0.35286	0.36255	
14		240	5.08	314.21	--	--	0.35372	--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
ents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.5650	0	6	310.11	--	--	0.33559	--	0.37036
3	(0.799 pk)	20	5.778	311.03	4.047E-02	4.021E-02	0.33959	0.37980	0.37036
4		40	5.616	311.73	3.121E-02	3.101E-02	0.34265	0.37366	
5		60	5.492	312.28	2.569E-02	2.552E-02	0.34508	0.37060	
6		80	5.387	312.75	2.261E-02	2.247E-02	0.34719	0.36966	
7		100	5.295	313.18	1.990E-02	1.977E-02	0.34909	0.36887	
8		120	5.217	313.55	1.776E-02	1.765E-02	0.35074	0.36839	
9		140	5.146	313.89	1.635E-02	1.624E-02	0.35227	0.36851	
10		160	5.082	314.20	1.499E-02	1.489E-02	0.35368	0.36857	
11		180	5.024	314.49	1.382E-02	1.373E-02	0.35497	0.36870	
12		200	4.971	314.76	1.273E-02	1.265E-02	0.35617	0.36882	
13		220	4.923	315.00	1.122E-02	1.114E-02	0.35728	0.36842	
14		240	4.883	315.20	--	--	0.35821	--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
ents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.6590	0	6	310.11	--	--	0.33559	--	0.38283
3	(0.932 pk)	20	5.711	311.31	5.209E-02	5.176E-02	0.34084	0.39260	--
4		40	5.511	312.19	4.072E-02	4.046E-02	0.3447	0.38516	
5		60	5.346	312.94	3.516E-02	3.493E-02	0.34803	0.38297	
6		80	5.207	313.60	3.176E-02	3.156E-02	0.35095	0.38251	
7		100	5.08	314.21	2.734E-02	2.716E-02	0.35372	0.38088	
8		120	4.984	314.69	2.428E-02	2.412E-02	0.35588	0.38000	
9		140	4.887	315.18	2.339E-02	2.324E-02	0.35811	0.38135	
10		160	4.802	315.63	2.072E-02	2.059E-02	0.36013	0.38071	
11		180	4.729	316.01	1.938E-02	1.926E-02	0.36189	0.38115	
12		200	4.657	316.40	1.835E-02	1.823E-02	0.36368	0.38191	
13		220	4.594	316.75	1.672E-02	1.661E-02	0.36527	0.38188	
14		240	4.536	317.07	--	--	0.36676	--	

	Irms(Y)	te(Y)	RNTC(X)	T(Y)	dTdt(Y)	Pc(Y)	Pradp(Y)	Ptotp(Y)	Ptotpave(Y)
Units	A	A	Ohm	K	K/s	W	W	W	W
ents					dT/dt	m*c*dT/dt	e*A*sB*T^4	Pc+Prad'	Ptot'ave
1				--	--	--	--	--	
2	0.7260	0	6	310.11	--	--	0.33559	--	0.40105
3	1.027 pk	20	5.636	311.64	6.927E-02	6.882E-02	0.34227	0.41109	--
4		40	5.36	312.88	5.655E-02	5.619E-02	0.34775	0.40394	
5		60	5.144	313.90	4.835E-02	4.804E-02	0.35231	0.40035	
6		80	4.96	314.81	4.327E-02	4.299E-02	0.35642	0.39942	
7		100	4.801	315.63	3.879E-02	3.854E-02	0.36015	0.39869	
8		120	4.664	316.36	3.526E-02	3.504E-02	0.3635	0.39854	
9		140	4.541	317.04	3.245E-02	3.225E-02	0.36663	0.39888	
10		160	4.432	317.66	2.994E-02	2.975E-02	0.3695	0.39925	
11		180	4.333	318.24	2.808E-02	2.790E-02	0.3722	0.40010	
12		200	4.242	318.78	2.574E-02	2.557E-02	0.37476	0.40033	
13		220	4.163	319.27	2.401E-02	2.386E-02	0.37704	0.40090	
14		240	4.087	319.75	--	--	0.3793	--	

Table 13. *Data for R_{LOAD} determination of the SS410 pan using various currents.*

The properties of the SS410 pan:

Quantity	**Symbol**	**Values**
Emissivity	e	0.8
Mass density	ρ	7700 kg/m^3
Heat capacity	c	464.7 J/kg.K
Heat capacity reference	c_{REF}	460 J/kg.K
Radiation area	A	2x2cmx2cm = 8x10^{-4} m^2
Volume	V	2cmx2cmx0.7mm=2.8x10^{-7} m^3
Mass	m	ρV=2.16x10^{-3} kg
Ambient temperature	T_0	303.66 K (R_{NTC0} = 7.864 kΩ)

Table 14. Properties of the SS410 pan

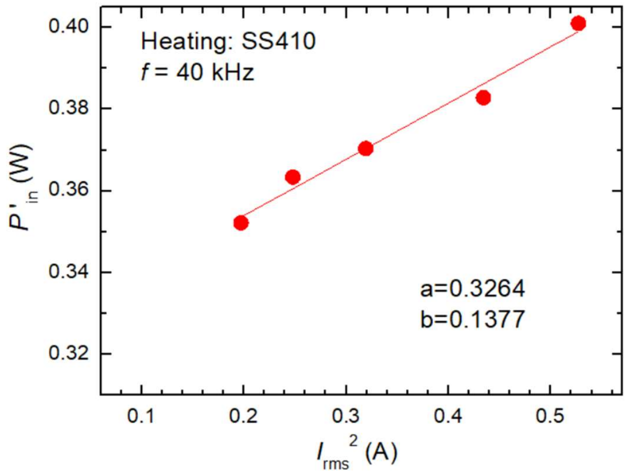


Figure 14. *"Effective load resistance" measurement of the SS410 pan.*

We then plot $P_{tot'ave}$ vs. I_{rms}^2 as shown above. The slope directly yields the load resistance R_{LOAD} = 137.7 mΩ which is 2.5x than that of the Al pan.

3.7 Better cooking pan: (b) SS410.
SS410 has significantly larger R_{LOAD} (2.5x) than that of Al, thus it is more efficient to be used as induction cooking pan.

3.8 Dominant physical parameter: (b) Magnetic permeability
SS410 is a magnetic stainless steel with very high permeability μ_r = 700, thus it has very small skin depth according to Eq. (16). Therefore, its R_{LOAD} is high and becomes more efficient for "cooking".

3.9 Induction cooker efficiency:

$$\eta = \frac{P_{IND-COOK}}{P_{IN}} = \frac{I_{rms}^{2} R_{LOAD}}{I_{rms}^{2}(R_{LOAD}+R_L)} = \frac{R_{LOAD}}{R_{LOAD}+R_L} \qquad (25)$$

From Q1.5 we have R_L = 0.48 Ω, we obtain: η_{Al} = 10.2% and η_{SS410} = 23.4%. Therefore the SS410 metal is more efficient to be used as induction cooking pan.

In summary for induction cooker, we want high conductivity to allow large eddy current to be generated but very small skin-depth that could be obtained in magnetic (high permeability) material to yield higher load resistance.

Appendix:

A. Alternative Solution to skin depth analysis

For skin depth experiment we can also analyze the problem into a single linear regression analysis instead of two as shown below:

$$\ln\left[\ln\left(\frac{V_i}{V_{i+1}}\right)\right] = -\frac{n}{2}\ln f + \frac{1}{2}\ln(\pi\sigma\mu t_0^{\,2}) \tag{26}$$

where *i* is the index of plate used in the experiment. The conductivity can be obtained from the linear regression intercept *a*:

$$\sigma = \exp(2a)/\pi\mu t_0^{\,2} \tag{27}$$

So essentially the student can perform the experiment with a single plate addition. An example of data is shown below:

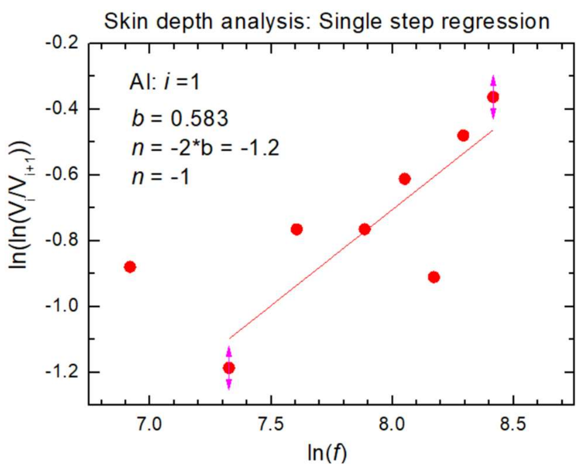


Figure 15. *Single regression analysis for skin depth investigation.*

The student can perform just two measurements e.g. V_2 with metal N=1 and N=2. We could also obtain n =-1.2 ~ -1, and σ = 1.0×10^7 S/m (-73% error from reference). We observe that this technique is less accurate as it utilize less data compared to double linear regression model that utilize e.g. N=5 x 5 frequencies data set.