

## Problem 1: The Earth's Horizontal Magnetic Field

### Section I

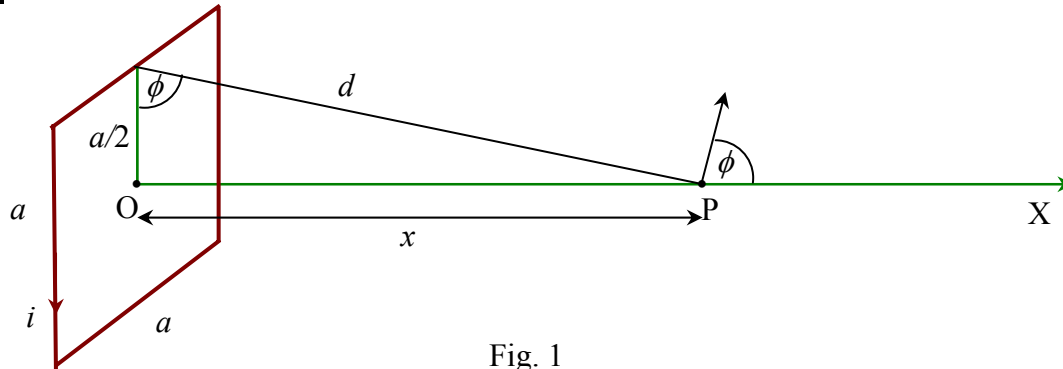


Fig. 1

At O, the centre of coil, the magnetic field for a single turn is

$$B_O = 4 \times \frac{\mu_0 i}{2\pi \left(\frac{a}{2}\right)} \frac{(a/2)}{\sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

At P, the horizontal magnetic field is

$$B_{Px} = 4 \frac{\mu_0 i}{2\pi d} \frac{(a/2)}{\sqrt{d^2 + \left(\frac{a}{2}\right)^2}} \cos \phi. \quad [0.3 \text{ point}]$$

From Fig. 1 we have  $d = \sqrt{x^2 + \left(\frac{a}{2}\right)^2}$  and  $\cos \phi = \frac{(a/2)}{\sqrt{x^2 + \left(\frac{a}{2}\right)^2}}.$  [0.2 point]

Then, for a square coil of  $N$  turns

$$B_{px} = \left(\frac{2\mu_0 i N}{\pi}\right) \cdot \frac{a/2}{\sqrt{x^2 + 2\left(\frac{a}{2}\right)^2}} \cdot \frac{a/2}{\left(x^2 + \left(\frac{a}{2}\right)^2\right)} \quad [0.2 \text{ point}]$$

or

$$B_{px} = \left(\frac{\mu_0 a^2 i N}{2\pi}\right) \left[ \frac{1}{\left(x^2 + \left(\frac{a}{2}\right)^2\right) \sqrt{x^2 + 2\left(\frac{a}{2}\right)^2}} \right] \quad [0.3 \text{ point}]$$

which becomes  $B_0 = \frac{2\sqrt{2}\mu_0 iN}{\pi a}$  as  $x = 0$ .

## **Section II**

Measurements to justify that we can ignore the torsion of the string.

length of string (cm)	time for 10 oscillations (sec)
2	9.38
4	9.69
6	9.90
8	10.13
10	10.13
12	10.22
14	10.12
25	10.12

(Note that this data is from a different magnet used in Section III.)

We can see that the period is constant for length of string  $\geq 10$  cm.

## **Section III**

The distance between the center of the magnet and the top surface of the platform for Part a), b) and c) is  $14.0 \pm 0.5$  cm.

### **a) Coil's magnetic field and Earth's horizontal magnetic field are in the same direction**

Since the coil's magnetic field ( $B$ ) and Earth's magnetic field ( $B_H$ ) are in the same direction, from

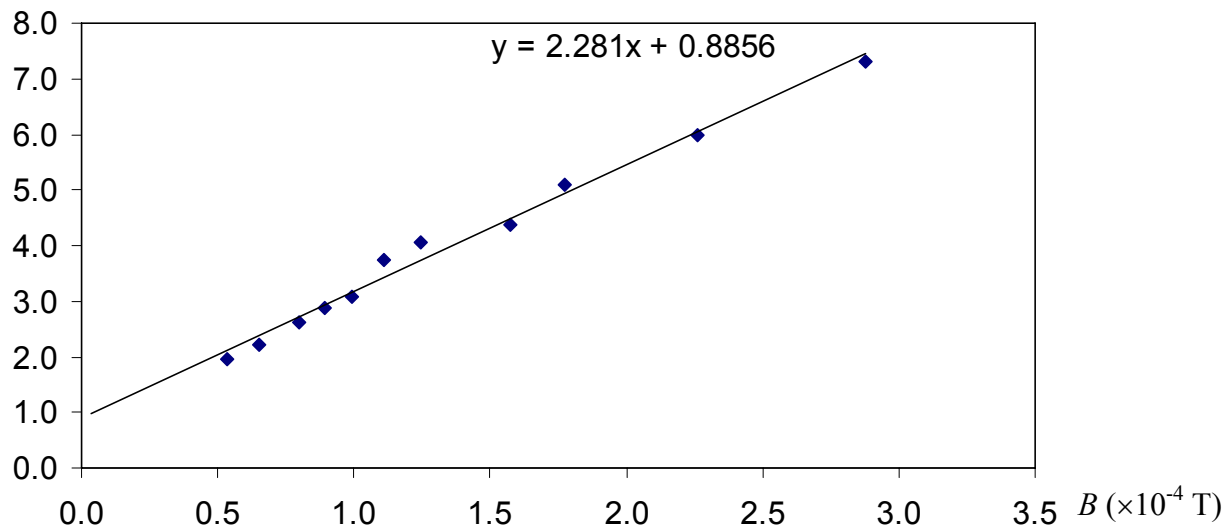
$$T = 2\pi\sqrt{\frac{I}{mB}} \quad \text{we have} \quad \frac{1}{T^2} = \beta B + \beta B_H \quad \text{where} \quad \beta = \frac{m}{4\pi^2 I} \quad \text{By plotting linear graph of } \frac{1}{T^2}$$

and  $B$  we can find  $B_H$  from its slope and intercept.

Measurement of 20 oscillations at different distances from coil, we get the result as in table.

$x$ (cm)	time for 20 oscillation (sec)		period $T$ (sec)	$B(\times 10^{-4} \text{ T})$	$1/T^2$
10	7.44	7.35	0.370	2.878	7.305
12	8.19	8.13	0.408	2.259	5.998
14	8.87	8.91	0.443	1.773	5.088
15	9.5	9.62	0.478	1.573	4.377
17	9.91	9.97	0.497	1.245	4.048
18	10.43	10.35	0.518	1.111	3.734
19	11.47	11.31	0.569	0.994	3.085
20	11.78	11.81	0.591	0.891	2.866
21	12.41	12.34	0.619	0.801	2.613
23	13.41	13.4	0.671	0.652	2.222
25	14.22	14.28	0.714	0.535	1.964

$1/T^2 \text{ (s}^{-2}\text{)}$



From graph we have: slope  $\beta = (2.281 \pm 0.063) \times 10^4 \text{ s}^{-2}/\text{T}$

intercept  $\beta B_H = 0.886 \pm 0.076 \text{ s}^{-2}$

The value of Earth's magnetic field is

$$B_H = \frac{0.8856}{2.281 \times 10^4} = 0.39 \times 10^{-4} \text{ T} = 0.39 \pm 0.04 \text{ G}$$

The magnetic moment of magnet is  $m = \beta^2 4\pi^2 M \left( \frac{L^2}{12} + \frac{r^2}{4} \right) = 1.68 \pm 0.09 \text{ A m}^2$

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**b) Earth's magnetic field only**

Time for 30 oscillations: 36.28, 36.25, 36.24 s.

Averaged period  $T_E = 1.209 \pm 0.001$  s

$$B_H = \frac{1}{T_E^2 \beta} = \frac{1}{1.21^2 \times 2.281 \times 10^{-4}} = 0.30 \pm 0.01 \text{ G}$$

**c) Coil's magnetic field and Earth's horizontal magnetic field are in opposite directions**The equilibrium (neutral) position  $x_0 = 31.0 \pm 0.2$  cm.

$$B_H = \left( \frac{\mu_0 a^2 i N}{2\pi} \right) \left[ \frac{1}{\left( x_0^2 + \left( \frac{a}{2} \right)^2 \right) \sqrt{x_0^2 + 2 \left( \frac{a}{2} \right)^2}} \right] = 0.31 \pm 0.01 \text{ G}$$

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